MATHEMATICS

Grade 7

Book 2

CAPS

Learner Book

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Chapter 1
Numeric and geometric patterns

In this chapter, you will learn to create, recognise, describe, extend and make generalisations about numeric and geometric patterns. Patterns allow us to make predictions. You will also work with different representations of patterns, such as flow diagrams and tables.

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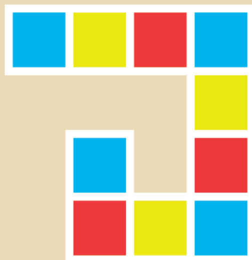

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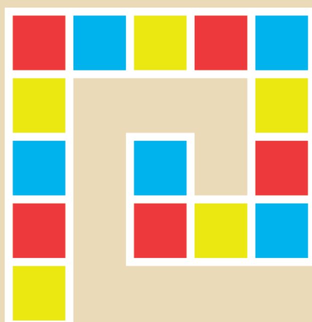

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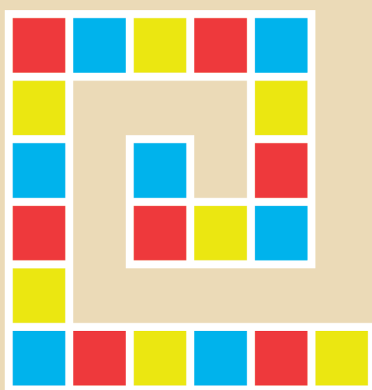

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1 Numeric and geometric patterns

1.1 Number patterns in sequences

WHAT COMES NEXT?

What may the next three numbers in each of these sequences be?

4; 8; 12; 16; 20; ....

4; 8; 16; 32; 64; ....

4; 8; 14; 22; 32; ....

5; 7; 4; 8; 3; 9; 2; ....

A set of numbers in a given order is called a **number sequence**. In some cases each number in a sequence can be formed from the previous number by performing the same or a similar action. In such a case, we can say there is a **pattern** in the sequence.

The numbers in a sequence are called the **terms** of the sequence. Terms that follow one another are said to be **consecutive**.

1. (a) Write down the next three numbers in each of these sequences:

   Sequence A: 4; 7; 10; 13; 16; ....

   Sequence B: 5; 10; 20; 40; 80; ....

   Sequence C: 2; 5; 10; 17; 26; ....

   (b) Write down how you decided what the next numbers would be in each of the three sequences.

....

....

....

A sequence can be formed by repeatedly adding or subtracting the same number. In this case the **difference** between one term and the next is constant.

A sequence can be formed by repeatedly multiplying or dividing by the same number. In this case the **ratio** between one term and the next is constant.

A sequence can also be formed in such a way that neither the difference nor the ratio between one term and the next is constant.

In sequence A of question 1 there is a **constant difference** between consecutive terms, as shown below.

Sequence A:	4	7	10	13	16
Difference:	+ 3	+ 3	+ 3	+ 3	

In sequence B of question 1 there is a **constant ratio** between consecutive terms, as shown below.

Sequence B:	5	10	20	40	80
Ratio:	× 2	× 2	× 2	× 2	

In sequence C of question 1 there is neither a constant difference nor a constant ratio between consecutive terms. There is, however, a pattern in the differences between the terms, which makes it possible to extend the sequence. Consecutive odd numbers, starting with 3, are added to form the next term.

Sequence C:	2	5	10	17	26
Difference:	+ 3	+ 5	+ 7	+ 9	

2. Write down the next five terms in each of the sequences below. In each case, describe the relationship between consecutive terms.
   (a) 100; 95; 90; 85; ....
   (b) 0,3; 0,5; 0,7; 0,9; ....
   (c) 6; 18; 54; 162; ....
   (d) 1; 3; 6; 10; 15; ....
   (e) 20; 31; 42; 53; ....
   (f) 10; 9,7; 9,4; 9,1; ....
   (g) 18 000; 1 800; 180; 18; ....

(h) $\frac{1}{48}$; $\frac{1}{24}$; $\frac{1}{12}$; $\frac{1}{6}$; ....

....

(i) 1; 4; 9; 16; ....

....

....

(j) 625; 125; 25; 5; ....

....

In all of the above cases it was possible to extend the sequence by repeatedly adding or subtracting a number to get the next term, or by repeatedly multiplying or dividing by a number to get the next term, or by adding different numbers according to some pattern to get the next term.

The word "recur" means "to happen again". The extension of a number sequence by repeatedly performing the same or similar action is called **recursion**. The rule that describes the relationship between consecutive terms is called a **recursive rule**.

RELATIONSHIPS BETWEEN DEPENDENT AND INDEPENDENT VARIABLES

1. (a) Mr Twala pays a fee to park his car in a parking lot every day. He has to pay R3 to enter the parking lot and then a further R2 for every hour that he leaves his car there. Complete the table below to show how much his parking costs him per day for various numbers of hours.

Number of hours	1	2	3	4	5	6	7	8	9
Cost of parking in R	5	7	9						

(b) How did you complete this table? Describe your method.

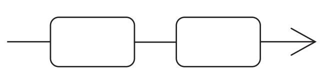
....

....

(c) Is there another way that you could complete the table? Describe it.

....

(d) Thembi multiplied the number of hours by 2 and then added 3 to calculate the cost for any specific number of hours. Complete the flow diagram to show Thembi's rule.



The rule *multiply by 2 and then add 3* describes the relationship between the two variables in this situation. The number of hours is the **independent variable**. The cost of Mr Twala's parking is the **dependent variable** because the amount he has to pay *depends on* the number of hours that he parks.

The R3 that is added is a **constant** in this situation. The number of hours and the cost are **variables**.

This rule describes how you can calculate the value of the *dependent* variable if the corresponding value of the *independent* variable is known. It differs from a recursive rule, which describes how you can calculate the value of the *dependent* variable that follows on a given value of the *dependent* variable.

In the case of a number sequence, the **position** (number) of the term can be taken as the independent variable, as shown for the sequence 15; 19; 23; 27; 31; . . . in this table:

Term number	1	2	3	4	5	6	7	8	50
Term	15	19	23	27	31				

2. (a) Complete the above table.
   (b) How did you calculate term number 50?

   ....

   ....

   ....

   (c) Lungile reasoned like this:
   I added 4 each time to complete the table. I counted backwards to see what comes before term 1. I got 11 and then I knew I had to add one 4 to 11 to get the first term.

   Lungile remembered that multiplication is done before addition, unless otherwise indicated by brackets.

   Complete the pattern below to show Lungile's thinking:

   Term 1: $11 + 1 \times 4 = 11 + 4 = 15$

   Term 2: $11 + 2 \times 4 = 11 + 8 = 19$

   Term 3: ....

   Term 4: ....

   Term 5: ....

   Term 6: ....

   Term 10: ....

   Term 50: ....

(d) Describe in your own words how term number 50 can be calculated.

....

(e) Tilly reasoned like this: *The constant difference between the terms is 4. I must add four 49 times to the first term to get the 50th term. So,* $15 + 49 \times 4 = 15 + 196 = 211$*.*
Complete the pattern below to demonstrate Tilly's thinking:

Term 1: 15

Term 2: $15 + 1 \times 4 = 15 + 4 = 19$

Term 3: $15 + 2 \times 4 = 15 + 8 = 23$

Term 4: ....

Term 5: ....

Term 6: ....

Term 10: ....

Term 50: ....

(f) Write the rule to calculate term number 50 in your own words.

....

In the example in question 2, the term number is the independent variable and the term itself is the dependent variable. So, if we know the rule that links the dependent variable and the independent variable, we can use it to determine any term for which we know the term number.

3. Write a rule to calculate the term for any term number in the sequence 15; 19; 23; 27; 31; . . . by using

(a) Lungile's thinking.

....

(b) Tilly's thinking.

....

We can use n as a symbol for "any term number".
The rule to calculate the term for any term number when using Lungile's thinking will then be:

Term = $n \times 4 + 11$

(c) Write down the rule to calculate the term for any term number in terms of n by using Tilly's thinking.

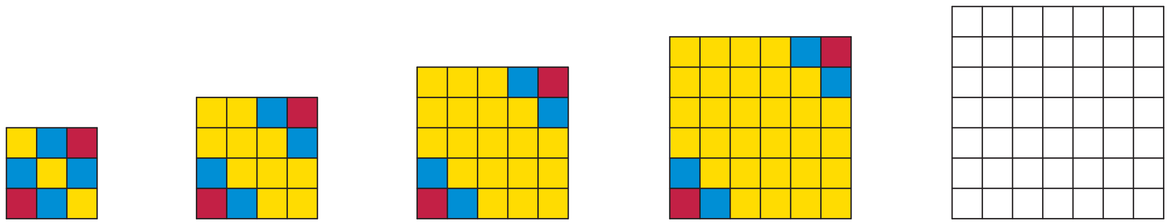
....

....

1.2 Geometric patterns

CONSTANT QUANTITIES AND VARIABLE QUANTITIES

Small yellow, blue and red tiles are combined to form larger square tiles as shown below:



Tile no. 1 *Tile no. 2* *Tile no. 3* *Tile no. 4* *Tile no. 5*

1. Draw tile no. 5 on the grid provided. (Shade the blue and red tiles in different ways. You don't have to use colours.)
2. Complete the table.

	Tile no. 1	Tile no. 2	Tile no. 3	Tile no. 4	Tile no. 5	Tile no. 10
Number of yellow tiles						
Number of red tiles						
Number of blue tiles						

3. How many red tiles are there in each bigger tile? ....
4. How many yellow tiles are there in each bigger tile?

....

....

5. Some of the quantities in this situation are variables and some are constants. Which are variables and which are constants?

....

....

6. Was it possible to predict the pattern on tile no. 2 by looking only at tile no. 1?

....

....

The number of red tiles is constant and the number of blue tiles is constant. It is clear that the design is such that there is always a red tile in the top right corner, and also in the bottom left corner, and that the red tiles are always "bordered" by two blue tiles each. So the number of red and blue tiles is **constant** in this situation.

The number of yellow tiles in the arrangements varies. The number of yellow tiles is a **variable** in this situation.

PATTERNS WITH MATCHES

1. A pattern with matches is shown below.

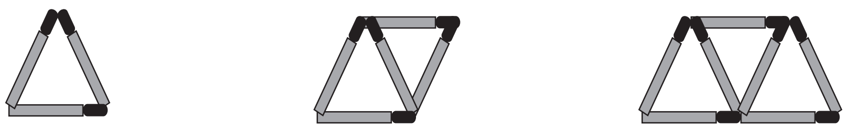


Figure 1 *Figure 2* *Figure 3*

(a) Explain how the pattern is formed.

....

....

(b) Complete the table.

Figure number	1	2	3	4	5	6	7	8
Number of matches	3	5	7					

(c) What rule did you use to complete the table?

....

(d) How many matches are needed to form figure no. 9? ....

(e) How many matches are needed to form figure no. 17? Explain.

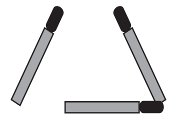
....

....

(f) If you used the recursive rule to complete the table, it would have taken a long time to answer question (e) because you had to add the same number many times. Try to find an easier way to answer question (e). Describe your method.

....

(g) Complete the pattern below.
Hint: It may help to think of figure no. 1 or term 1 like this: There is 1 match at the beginning and two more are added every time. It helps to "see" the two matches that are added each time.



Term 1: $1 + 1 \times 2 = 3$

Term 2: $1 + 2 \times 2 = 5$

Term 3: $1 + 3 \times 2 = 7$

Term 4: ....

Term 5: ....

Term 10: ....

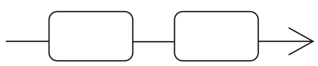
Term 17: ....

(h) What stays the same in the pattern in (g) and what varies?

....

....

(i) Use the flow diagram below to write down the rule that you can use to calculate the number of matches needed for any figure in the pattern.

Figure number  *Number of matches*

(j) Can you link the number of matches added each time to the number that you multiply by in the flow diagram? Explain.

....

....

2. Another pattern with matches is shown below.

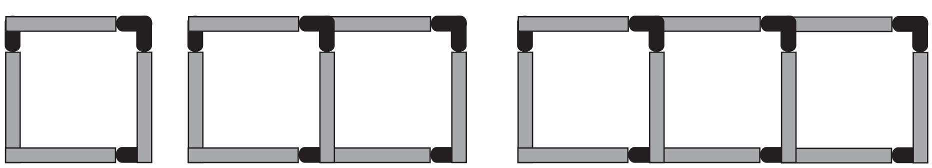


Figure 1 *Figure 2* *Figure 3*

(a) Explain how the pattern is formed.

....

....

(b) Complete the table.

Figure number	1	2	3	4	5	6	7	8
Number of matches	4							

(c) What rule did you use to complete the table?

....

(d) How many matches are needed for figure 9 (or term 9)? ....

(e) How many matches are needed for figure 20 (or term 20)? ....

(f) What rule did you use to calculate the number of matches in question (e)?

....

(g) Complete the pattern:

Term 1: $1 + 1 \times 3 = 4$

Term 2: $1 + 2 \times 3 = 7$

Term 3: $1 + 3 \times 3 =$ ....

Term 4: ....

Term 5: ....

Term 10: ....

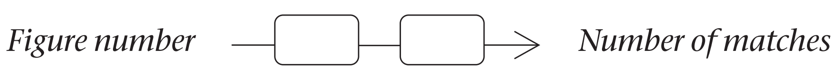
Term 17: ....

(h) What stays the same in the pattern in (g) and what varies?

....

....

(i) Use the flow diagram below to write down the rule that you can use to calculate the number of matches needed for any figure in the pattern.



3. Compare the way in which the number of matches increases in question 1 to the way in which it increases in question 2. What is the same and what is different?

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....

....

ALPHABETIC PATTERNS

Consider the figures below formed with red dots.

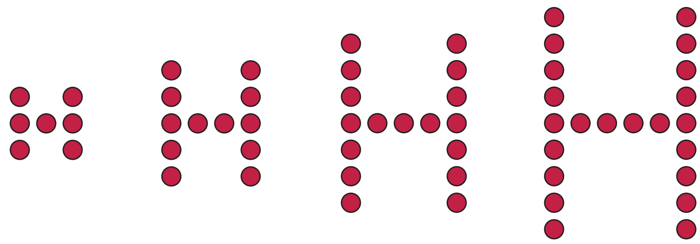


Figure 1 *Figure 2* *Figure 3* *Figure 4* *Figure 5*

1. How many dots are used to form figure 5?

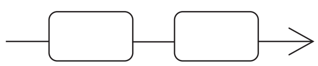
   ....

2. Draw figure 5.

3. Complete the table.

Figure number	1	2	3	4	5	6	7	8
Number of dots	7	12	17					

4. Complete the flow diagram.



5. What rule did you use to complete the table? Describe your rule.

   ....

6. Can you think of another rule to complete the table? Describe your rule.

   ....

   ....

7. Name the dependent variable and the independent variable in this situation.

   ....

   ....

SQUARES AND CUBES

1. Squares are arranged to form figures as shown below, according to a rule.

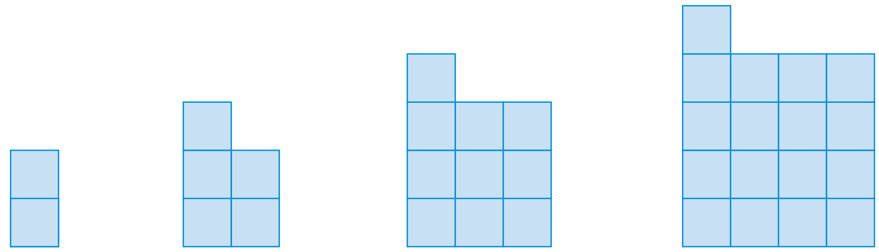


Figure 1 *Figure 2* *Figure 3* *Figure 4*

(a) Complete the table. Then determine the differences between consecutive terms.

Figure number	1	2	3	4	5	6	7	8
Number of squares	2	5						

+ 3

(b) Describe the recursive rule that you can use to extend the pattern in words.

....

(c) Nombuso played around with the differences between consecutive terms. She noticed that the pattern (+ 3; + 5; + 7; ...) was similar to the one that you get when you calculate the differences between square numbers. This made her think that she should investigate square numbers to help her find a rule that could link the figure number and the number of squares.

Complete the following pattern along the lines of Nombuso's thinking:

Figure 1: $1 \times 1 + 1 = 1 + 1 = 2$

Figure 2: $2 \times 2 + 1 = 4 + 1 = 5$

Figure 3: ....

Figure 4: ....

Figure 5: ....

Figure 6: ....

Figure 7: ....

Figure 8: ....

Figure 50: ....

(d) Write a rule to calculate the number of squares for any figure number.

....

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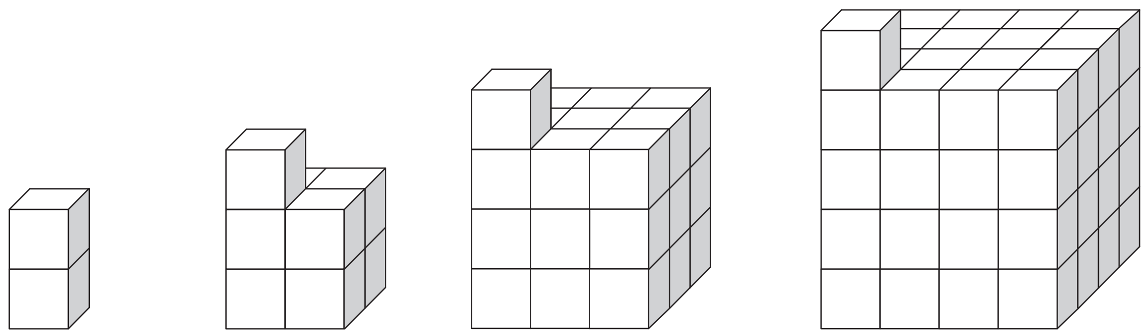
(e) Write your rule in (d) in terms of *n* where *n* is the symbol for any figure number.

(f) Compare the sequence in this activity to the sequence in the previous activity where dots were arranged to form the letter H. Describe the way in which the dependent variable (the output number) changed in each of the sequences.

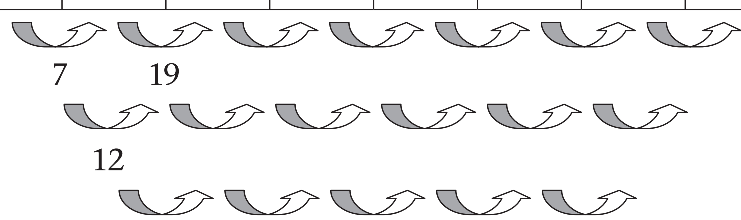
2. Identical cubes are arranged to form stacks of cubes in the following way:



Stack 1 *Stack 2* *Stack 3* *Stack 4*

(a) Complete the table. Then find the differences between consecutive terms. Do it a second and a third time. Write the differences below the arrows.

Stack number	1	2	3	4	5	6	7	8
Number of cubes	2	9	28					



(b) Describe the way in which you completed the table.

(c) David looked carefully at the structure of the stacks and did the following to link the stack number with the number of cubes in a stack. Complete the pattern.

Stack 1: $1 \times 1 \times 1 + 1 = 1 + 1 = 2$

Stack 2: $2 \times 2 \times 2 + 1 = 8 + 1 = 9$

Stack 3: $3 \times 3 \times 3 + 1 = 27 + 1 = 28$

Stack 4: $4 \times 4 \times 4 + 1 = 64 + 1 = 65$

Stack 5: ....

Stack 6: ....

Stack 7: ....

Stack 8: ....

Stack 9: ....

Stack 10: ....

(d) How many cubes will there be in stack 50?

....

....

(e) Write the rule that you used to calculate the number of cubes in stack 50 in words.

....

(f) Write your rule in (e) in terms of *n* where *n* is the symbol for any stack number.

....

3. In questions 1(a) and 2(a) you calculated the differences between the consecutive terms.

   (a) What did you find when you kept on finding the differences, as suggested in question 2(a)?

   ....

   (b) Go back to question 1(a). What do you find when you keep on finding the differences between consecutive terms, like you did in question 2(a)?

   ....

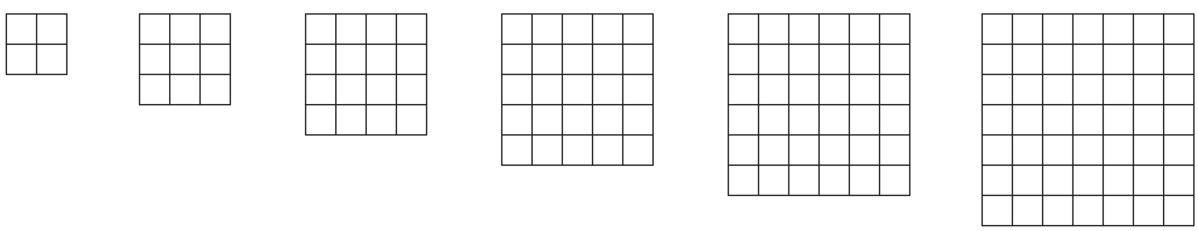
   ....

MY OWN PATTERNS

Use the grid, the tables and the tile template to create and describe your own geometric patterns.

Pattern A

Pattern B



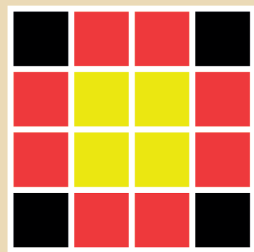
Chapter 2
Functions and relationships 1

In this chapter, you will work with formulae. A formula is a description of how one may calculate the value of one variable quantity if the value or values of certain other variable quantities are given.

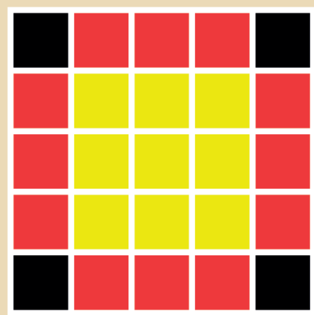
You will acquire the symbolic language to write formulae with two variables.

1 yellow $\rightarrow$ 4 red

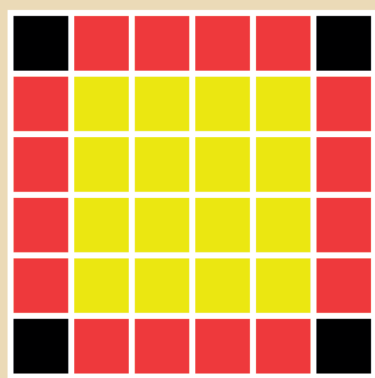
4 yellow $\rightarrow$ 8 red



9 yellow $\rightarrow$ 12 red



16 yellow $\rightarrow$ 16 red

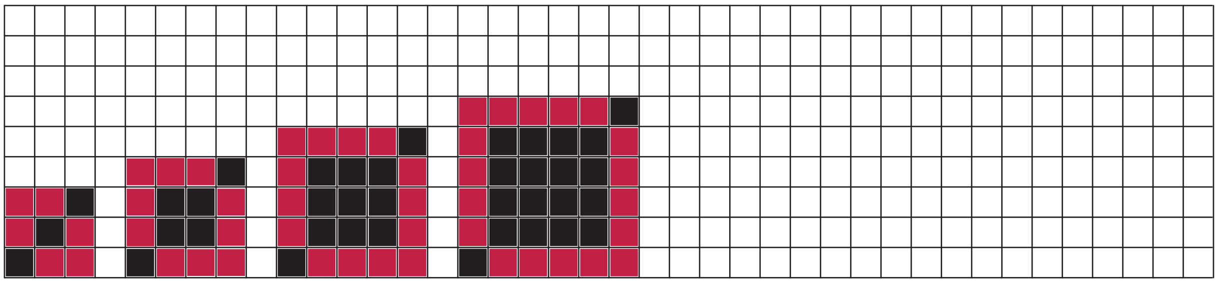


25 yellow $\rightarrow$? red

2 Functions and relationships 1

2.1 From counting to calculating

1. (a) How many red squares and how many black squares are there in each of the arrangements 1, 2, 3 and 4 below? Write your answers in the table.



Arrangement number	1	2	3	4			
Number of red squares							
Number of black squares							

(b) Imagine that arrangements 5, 6 and 7 are made according to the same pattern. How many red and how many black squares do you think there will be in each of these arrangements? Write your answers in the above table.

(c) Draw arrangements 5 and 6 on the above grid, if you have not done so already.

(d) Try to figure out how many red and how many black squares there will be in arrangements 20, 21 and 22.

....

....

....

2. It will be useful to have formulae to calculate the numbers of red and black squares in different arrangements like the above.

(a) Which of the formulae below can be used to calculate the numbers of red squares in the above arrangements? There is more than one formula that works.

$y = 2 \times x + 4$ $\quad$ $y = 2 \times (2 \times x + 1)$ $\quad$ $y = x^2 + 2$ $\quad$ $y = 4 \times x + 2$

....

(b) Natasha decided to use the formula $y = 4 \times x + 2$ to calculate the number of red squares in an arrangement. What do the symbols x and y mean in this case?

....

(c) Use the formula $y = 4 \times x + 2$ to calculate the numbers of red squares in arrangements 20, 21 and 22.

....

(d) If your answers differ from the answers you gave in question 1(d), you have made mistakes somewhere. Find your mistakes and correct them.

(e) Complete the table.

x	1	2	3	4	5	6	7	8	9
$2 \times (2 \times x + 1)$									
$2 \times x + 4$									
$4 \times x + 2$									

3. (a) Which of the formulae below can be used to calculate the numbers of black squares in the arrangements in question 1?

$z = (x + 2)^2$ $z = x^2 + 2$ $p = n^2 + 2$

....

(b) Complete the table.

x	1	2	3	4	5	6	7	8	9
$x^2 + 2$									
$(x + 2)^2$									

4. Hilary uses x to represent the *number of squares in each side* of the arrangements.

(a) Which of these formulae can Hilary use to calculate the numbers of black squares in the arrangements in question 1?

$y = x^2 - 4 \times x + 6$ $y = (x - 2)^2 + 2$ ....

(b) Which of these formulae can Hilary use to calculate the numbers of red squares?

$y = 3 \times x - 3$ $y = 4 \times x - 6$ $4 \times (x - 2) + 2$ ....

(c) Complete this table to check your answers.

x	3	4	5	6	7	8	9
$x^2 - 4 \times x + 6$							
$(x - 2)^2 + 2$							
$3 \times x - 3$							
$4 \times x - 6$							
$4 \times (x - 2) + 2$							

2.2 What to calculate and how

REPRESENTING SITUATIONS MATHEMATICALLY

1. (a) How many minutes are there in an hour? ....

   (b) How many minutes are there in 2 hours? ....

   (c) How many minutes are there in 3 hours? ....

   (d) Explain how you determined the answers for questions 1(b) and (c).

   ....

   ....

The formula $m = 60 \times h$ can be used to calculate the number of minutes when the number of hours is known. The symbol h represents the number of hours and m the number of minutes.

   (e) Express the formula $m = 60 \times h$ in words.

   ....

   (f) Complete the table.

Number of hours	1	2	3	15	24
Number of minutes	60	120			
How to calculate	60×1				

2. Three bus companies placed the following advertisements in a newspaper:

   (a) Which of the formulae at the top of the next page can be used to calculate the fare for a journey with Hamba Kahle Tours?

   ....

   (b) Which of the formulae can be used to calculate the fare for a journey with Saamgaan Tours?

   ....

Saamgaan Tours
We criss-cross every province and stop in every town and dorpie. Pay only R450 per trip plus 60c per km.

Hamba Kahle Tours
Long distance travel is our business: R500 per trip plus 50c per km!

Comfort Tours
Experience what it means to travel in style. Only R480 per trip plus 55c per km.

Some formulae to calculate fares:

A. Fare = 0,50 × distance + 500

B. Fare = 50 × distance + 500

C. Fare = 0,60 × distance + 450

D. Fare = 60 × distance + 450

E. Fare = 55 × distance + 480

F. Fare = 0,55 × distance + 480

(c) Which of the above formulae can be used to calculate the fare for a journey with Comfort Tours?

We write 50c as R0,50 or 0,50 when we do calculations.

....

(d) Complete the table by making use of the formulae below. You may use a calculator for this question.

Fare for Hamba Kahle Tours = 0,50 × distance + 500

Fare for Saamgaan Tours = 0,60 × distance + 450

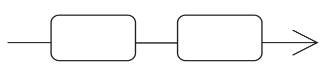
Fare for Comfort Tours = 0,55 × distance + 480

Distance in km	150	200	250	300
Hamba Kahle Tours				
Saamgaan Tours				
Comfort Tours				

(e) Which bus company is the cheapest? Explain.

....

(f) Complete a flow diagram for the bus company that you named in question (e):



(g) Wandile wrote the formulae for calculating the fares for the different bus companies using the letter symbols x and y. Say what each letter symbol stands for in each of the following:

(i) $y = 0{,}50 \times x + 500$ ....

(ii) $y = 0{,}60 \times x + 450$ ....

(iii) $y = 0{,}55 \times x + 480$ ....

(h) Which of the three bus companies would be the cheapest to use for a journey of 1 000 km?

....

....

2.3 Input and output numbers

FROM FORMULAE TO TABLES

1. For each of the tables, determine which of these formulae could have been used to complete it:

A. $y = 5 \times x + 3$ B. $y = 3 \times x$ C. $y = 3 \times x + 2$
D. $y = 4 \times x$ E. $y = 3 \times x + 1$ F. $y = 2 \times x$
G. $y = 3 \times x + 10$ H. $y = 2 \times x - 1$ I. $y = 5 \times x$

(a)

x	1	2	3	4	5
y	13	16	19	22	25

Formula used: ....

(b)

x	1	2	3	4	5
y	8	13	18	23	28

Formula used: ....

(c)

x	1	2	3	4	5
y	4	8	12	16	20

Formula used: ....

(d)

x	1	2	3	4	5
y	5	8	11	14	17

Formula used: ....

(e)

x	1	2	3	4	5
y	5	10	15	20	25

Formula used: ....

(f)

x	1	2	3	4	5
y	1	3	5	7	9

Formula used: ....

We can complete a table of values if we are given a formula. For example, for the formula $y = 7 \times x - 3$ we can complete the table below, as shown:

For $x = 1$, $y = 7 \times 1 - 3$
$= 7 - 3$
$= 4$

For $x = 2$, $y = 7 \times 2 - 3$
$= 14 - 3$
$= 11$

For $x = 3$, $y = 7 \times 3 - 3$
$= 21 - 3$
$= 18$

x	1	2	3	4	5	6	7
y	4	11	18	25	32	39	46

2. Use the given formulae to complete the tables.

(a) $y = 6 \times x - 5\frac{1}{2}$

x	1	2	3	4	5	6	7
y							

(b) $y = 30 \times x + 1$

x	0,1	0,2	0,3	0,4	0,5	0,6	0,7
y							

FROM PATTERNS TO FORMULAE

1. Some arrangements with black and red squares and some formulae are given below.

Formula A: $z = 2 \times n + 1$
Formula B: $z = 2 \times x - 3$
Formula C: $y = (n + 1)^2 + 2$
Formula D: $y = x^2 - (2 \times x - 3)$

(a) How many black squares will there be in the next two similar arrangements?

....

(b) Susan uses formulae B and D to calculate the numbers of red and black squares. What do the letter symbols z, x and y mean in Susan's work?

....

....

(c) Zain uses formulae A and C to calculate the numbers of red and black squares. What do the letter symbols z, n and y mean in Zain's work?

....

....

2. Write formulae that can be used to calculate the numbers of red and black squares in arrangements like those below. Use letter symbols of your own choice and state clearly what each of your symbols represents.

....

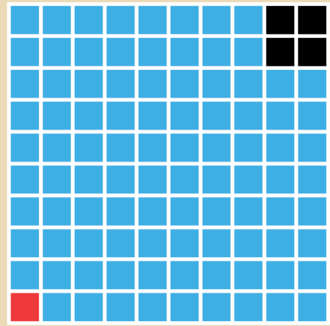
....

Chapter 3
Algebraic expressions 1

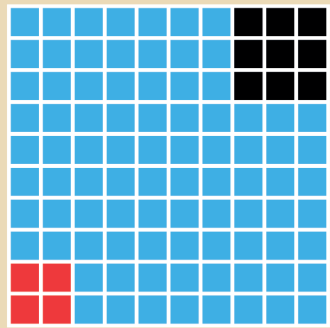
In this chapter, you will learn about algebraic expressions. An algebraic expression is a computational procedure. Put differently, an algebraic expression tells you how to calculate a value. But an algebraic expression *is* also a value.

You will learn more about variable and constant quantities in this chapter and you will be required to identify these in formulae and number sentences.

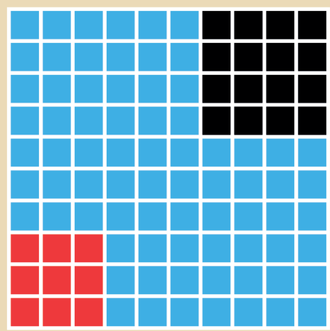
$100 - 1^2 - 2^2$



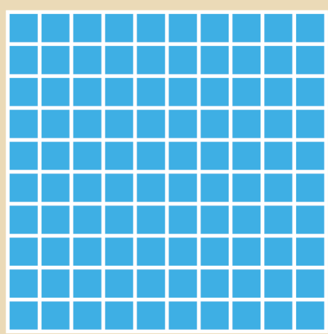
$100 - 2^2 - 3^2$



$100 - 3^2 - 4^2$



$100 - x^2 - (x + 1)^2$



3 Algebraic expressions 1

3.1 Describing and doing computations

DIFFERENT WAYS OF DESCRIBING A COMPUTATION

1. The diagrams below represent arrangements of small circles. In every arrangement there are two rows of circles.

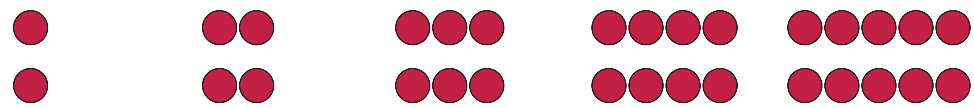


Diagram 1 *Diagram 2* *Diagram 3* *Diagram 4* *Diagram 5*

(a) The table below relates to the diagrams. Complete it.

Diagram number	**1**	**2**	**3**	**4**	**5**
Number of circles per row					
Number of rows					
How to calculate the total number of circles per diagram (**rule**)					

In every diagram, we can identify:

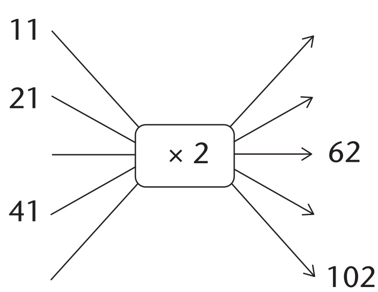
- the number of rows
- the number of circles per row
- the total number of circles per arrangement.

(b) What remains the same in the diagrams? ....

(c) What changes in the diagrams? In other words, what are the variable quantities in the situation?

....

(d) Complete the flow diagram.



(e) How many circles will diagram 11 have if the pattern is extended? Explain.

....

....

(f) What does the number 2 in the rule $2 \times n$ represent?

....

(g) What does the letter symbol n represent in the rule $2 \times n$?

....

....

The rule $2 \times n$ can be used to determine the total number of circles in a diagram. The number 2 in the rule $2 \times n$ remains the same all the time. We say it is a **constant**. n represents the number of circles per row and that is a **variable**, because it changes.

Consider the sequence 1; 3; 5; 7; 9; ...

The first odd number can be written as $2 \times \mathbf{1} - 1$.

The second odd number can be written as $2 \times \mathbf{2} - 1$.

The third odd number can be written as $2 \times \mathbf{3} - 1$.

The numbers 2 and –1 remain the same all the time; we call them **constants**. The numbers in blue change according to the position of the odd number in the sequence. We call them **variables**.

2. (a) What is the tenth odd number?

....

(b) What is the thirtieth odd number?

....

(c) What is the hundredth odd number? ....

(d) What is the nth odd number? ....

3. The rule $2 \times n - 1$ can be used to determine any odd number in the sequence 1; 3; 5; 7; 9; ...

What does the letter symbol n represent in the rule $2 \times n - 1$?

....

....

In the questions above we have used the letter symbol n to represent:

1. a changing number in the rule $2 \times n$ (n represents the number of circles in a row)
2. the position of the odd number in a sequence in the rule $2 \times n - 1$.

The rule $\mathbf{2 \times n}$ can be used to calculate the total number of circles in a diagram if the number of circles per row is known.

The rule $\mathbf{2 \times n - 1}$ can be used to determine any odd number in the sequence of odd numbers if its position is known.

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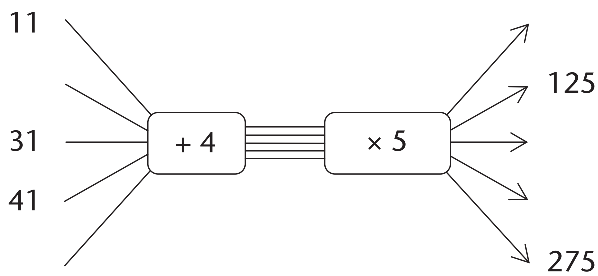
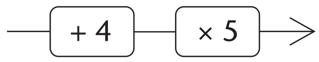
4. (a) Complete the flow diagram.



> We call the numbers on the left in the flow diagram the **input numbers**.
> The numbers on the right in the flow diagram, and whose values depend on the input numbers, are called the **output numbers**.

(b) Which of the following instructions did you follow to calculate the output values of  in question 4(a)?

Make a tick mark (✓) next to the correct answer.

A. Multiply the input number by 5 and then add 4.

B. Add 45 to the input number.

C. Add 4 to the input number and then multiply by 5.

5. Use 10 as input number and calculate the output number for each of the word formulae in question 4(b).

....

We may write $(x + 4) \times 5$ as an abbreviation for *add 4 to the input number, then multiply by 5.* $(x + 4) \times 5$ can be called a computational instruction or an **algebraic expression**.

> The letter symbol x, or any other symbol, can be used as an abbreviation for "the input number".

In the expression $(x + 4) \times 5$, the letter symbol x can be replaced by many different input numbers. The symbol x represents a **variable quantity** or a **variable**. If, however, the expression $(x + 4) \times 5$ is equal to 35, as in the number sentence $(x + 4) \times 5 = 35$, the symbol x represents only one value, and that is 3.

In the expression $(x + 4) \times 5$, the numbers 4 and 5 are **constants**. In the number sentence $(x + 4) \times 5 = 35$, x is an **unknown value**.

6. Write the abbreviations for the following computational instructions by using x for "the input number":

(a) Half the input number plus 2 ....

(b) Multiply the input number by 6 and subtract 2. ....

(c) Multiply the sum of the input number and 3 by 10. ....

(d) Subtract 4 from the input number and multiply the answer by 7. ....

7. Cardo's teacher writes on the board: "Add 2 and then multiply the answer by 3." The class must use 5 as an input number and apply the computational instruction.
   (a) Cardo uses 5 as the input number and writes: $(5 + 2) \times 3$.
   Paul says $(5 + 2) \times 3$ is 7×3 which is 21. Is Paul right? ....
   (b) Explain your answer in (a).

   ....

   ....

   ....
   (c) Represent this flow diagram as an algebraic expression:

   x → [+ 2] → [× 3] → ....

8. Express each computational instruction as a flow diagram and then write the abbreviation (algebraic expression) with x as input number:
   (a) Multiply by 4 and then subtract 8.
   → [ ] → [ ] → ....
   (b) Subtract 8 and then multiply by 4.
   → [ ] → [ ] → ....
   (c) Add 15 and then divide by 5.
   → [ ] → [ ] → ....
   (d) Divide by 5 and then add 15.
   → [ ] → [ ] → ....

9. Describe each computational instruction in words:
   (a) → [× 4] → [+ 7] → ....
   (b) → [+ 7] → [× 4] → ....
   (c) → [× 9] → [– 5] → ....
   (d) → [– 5] → [× 9] → ....

10. Two algebraic expressions are given in the table. Use the given input values (x values) to determine the corresponding output values.

x	1	2	3	4	5	6
$6 \times x + 8$	14	20	26			
$2 \times x \times (3 + 4)$						

3.2 Relationships represented in formulae

MAKING SENSE OF VARIABLES AND CONSTANTS IN FORMULAE

1. (a) Chris uses the formula $P = 2 \times l + 2 \times b$ to calculate the perimeters of rectangles of differing lengths and breadths as indicated in the table. He also calculates the area of each rectangle using the formula $A = l \times b$.
   Complete the table.

Rectangle	**1**	**2**	**3**	**4**
Length (l)	24	6	8	12
Breadth (b)	1	4	3	2
Perimeter $P = 2 \times l + 2 \times b$				
Area $A = l \times b$				

   (b) Rita calculates the perimeter of a rectangle in a different way. She adds the value of the length of the rectangle to the value of the breadth of the rectangle and then multiplies the answer by 2.
   Write down the formula that Rita uses to calculate the perimeter of each rectangle. Test whether or not Rita's formula produces the same results as Chris's.

   ....

   Questions 1(c) to (e) refer to the formula $P = 2 \times (l + b)$.

   (c) What does the number 2 represent in the formula?

   ....

   (d) What is the number 2 called? ....

   (e) Which letter symbols represent variables in the formula $P = 2 \times (l + b)$? Explain.

   ....

   ....

   (f) What can you say about the area of all of these rectangles?

   ....

2. Sindi calculates her father's age by using the formula $F = x + 37$, where x is Sindi's age. Her father passed away when Sindi was 43 years old. How old was he then?

   ....

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3. Jacob wants to buy the cheapest cell phone on the market. He has already saved R45 and decides to save R5 per week until he has enough money to buy the phone. The formula $y = 45 + 5 \times w$ gives the amount of money (in rands) that Jacob has saved to buy the cell phone after w weeks.

   (a) Complete the table. The first row has been done as an example.

Number of weeks (w)	How to calculate $45 + 5 \times w$	Amount saved (y)
0	$45 + 5 \times 0 = 45 + 0$	45
1		
2		
4		
5		

   (b) The cell phone that Jacob wants to buy costs R90. Will Jacob have saved enough money to be able to buy the cell phone by the eighth week? Explain.

....

....

   (c) Complete the table.

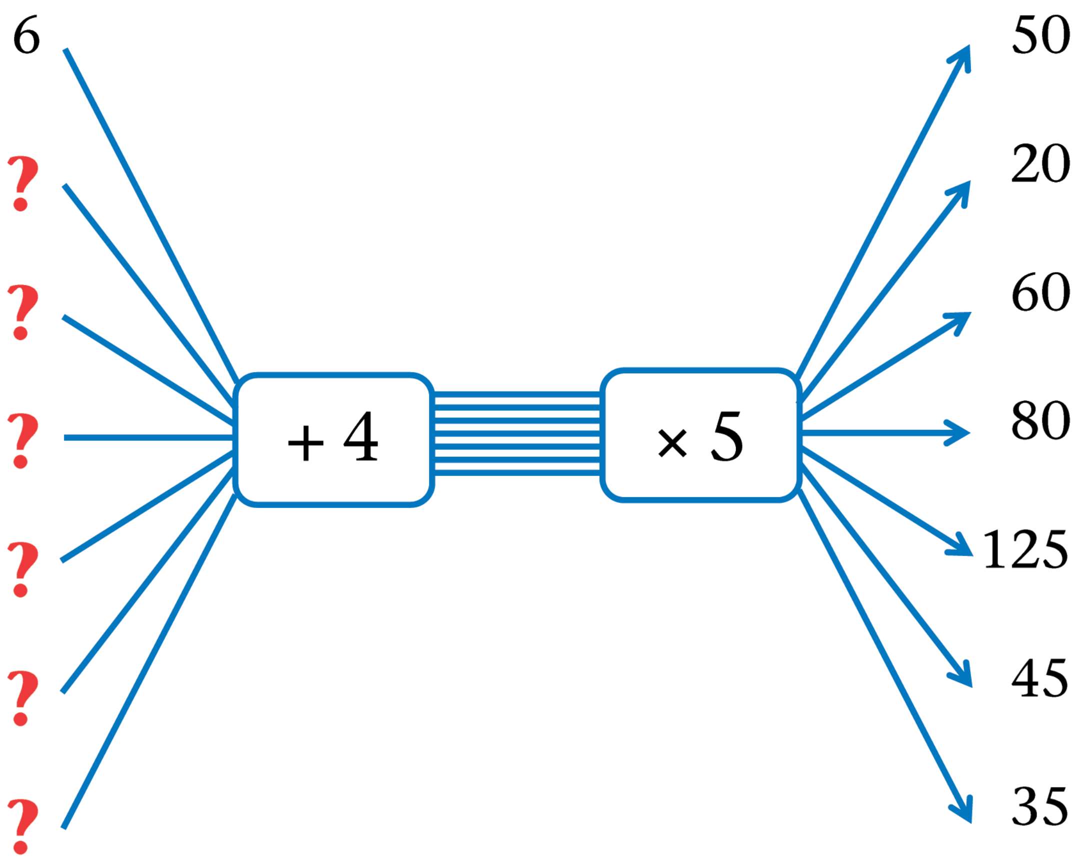
	Formula: $y = 45 + 5 \times w$	Explanation
Which are constants in the formula?		
Which letter symbols represent variable quantities in the formula?		

4. In each of the following formulae, identify the symbols that represent variables and constants.

		Symbols for variable(s)	Constant(s)
(a)	$y = 5 \times x + 7$		
(b)	$y = 100 + x$		
(c)	$y = x \div 5$		
(d)	$y = 5 \times x$		
(e)	$y = 0,7 \times x + 2,3$		

Chapter 4
Algebraic equations 1

In this chapter you will learn about solving open number sentences (or equations) by inspection and by the trial and improvement method. You will also represent problem situations by means of number sentences as well as analyse and interpret some number sentences.


6
?
?
?
?
?
?
+ 4
× 5
50
20
60
80
125
45
35

4 Algebraic equations 1

4.1 Solving by inspection

NUMBER PUZZLES

Solve these number puzzles.

1. I am thinking of a certain number. If I add 3 to that number, the answer is 13. What is the number?

   ....

2. I am thinking of a certain number. If I multiply that number by 5, the answer is 30. What is the number?

   ....

3. I am thinking of a certain number. If I multiply that number by 3 and then add 4 to the result, the answer is 19.
   (a) Is the number 3? Give a reason for your answer.

   ....

   (b) Is the number 4? Give a reason for your answer.

   ....

   (c) Is the number 5? Give a reason for your answer.

   ....

   (d) Is the number 6? Give a reason for your answer.

   ....

Number puzzles like those above can be shortened by using letter symbols as place holders for unknown numbers. In the case of question 1 we can write the following number sentence: $\boldsymbol{x + 3 = 13}$.

In the case of a number sentence such as $x + 3 = 13$ we cannot say whether it is true or false until we have determined the value of the unknown. The value of the unknown that makes the number sentence (an **equation**) true is called the **solution** of the number sentence.

For the number sentence $x + 3 = 13$, the solution is $x = 10$ because it makes the number sentence true.

A mathematical statement such as $x + 3 = 13$ that could be true or false depending on the value of x, is called an **open number sentence** or an **equation**.

To make a number sentence **true** means to find its **solution**.

THE SOLUTION IS THERE TO SEE

The solution to the number sentence $x + 4 = 20$ can be seen at once. The value of x is 16 simply because $16 + 4 = 20$. In this case, we say we solve the number sentence **by inspection**.

Solve these number sentences (equations) by inspection.

1. (a) $x - 8 = 8$ (b) $x + 7 = 20$

   ....

   (c) $\frac{16}{x} = 8$ (d) $\frac{x}{16} = 2$

   ....

   (e) $5 \times x = 40$ (f) $8 \times x = 40$

   ....

2. (a) $84 \div x = 7$ (b) $36 \div x = 4$

   ....

   (c) $x + 56 = 100$ (d) $100 - x = 56$

   ....

4.2 Solving by the trial and improvement method

Sometimes you cannot see the solution of a number sentence (an equation) at once. Look at the following number puzzle or equation, for example:

I am thinking of a number. 6 × *the number* – 11 = 43. What is the number?

In this case, you will have to try many different possible solutions until you identify the correct one. Here we can use a method known as **trial and improvement** to determine the solution. It is shown in the table below.

Possible solution	Test	Conclusion
Try 5	$6 \times 5 - 11 = 30 - 11 = 19$	5 is too small
Try 10	$6 \times 10 - 11 = 60 - 11 = 49$	10 is too big
Try 8	$6 \times 8 - 11 = 48 - 11 = 37$	8 is too small
Try 9	$6 \times 9 - 11 = 54 - 11 = 43$	9 is the solution

Solve the following equations by means of the trial and improvement method. In each case, the solution is a number between 1 and 20.

1. $2 \times x + 13 = 37$ The solution is $x =$ ....

Possible solution	Test	Conclusion

2. $14 \times x - 21 = 77$ The solution is $x =$ ....

Possible solution	Test	Conclusion

3. $7 \times x + 8 = 71$ The solution is $x =$ ....

Possible solution	Test	Conclusion

4. $4 \times x + 7 = 31$ The solution is $x =$ ....

Possible solution	Test	Conclusion

5. $10 \times x + 11 = 141$ The solution is $x =$ ....

Possible solution	Test	Conclusion

SOLVING BY INSPECTION OR TRIAL AND IMPROVEMENT

Solve the following equations by inspection or by the trial and improvement method:

1. (a) $x + 5 = 2 \times x$ (b) $k \times 5 = 20 + k$

....

....

....

(c) $2 \times q = 18 - q$ (d) $3 \times t = t + 22$

....

....

....

2. (a) $y + 6 = 4 \times y$ (b) $5 \times p = 18 + 2 \times p$

....

....

....

(c) $4 \times z = 18 + z$ (d) $x \times 5 = 20$

....

....

....

(e) $42 \div m = 35 - 29$ (f) $3 \times x - 2 = x + 6$

....

....

....

4.3 Describing problem situations with equations

FROM WORDS TO EQUATIONS

Write an equation using a letter symbol as a placeholder for the unknown number to describe the problem in each of the situations below.

1. There are 30 learners in a class. x learners are absent and 19 are present.

   ....

2. There are 70 passengers on a bus. At a bus stop m passengers get off. There are now 23 passengers on the bus.

   ....

3. A boy buys a bicycle for R1 260 on lay-by. How many payments of R90 each must he make to pay for the bicycle? Let x be the number of payments to be made.

   ....

4. Five people share a total cost of R240 equally amongst themselves. Let c be the cost per person.

   ....

5. A school charges R100 a day for the use of its training facilities for athletes plus R30 per athlete per day for food and use of equipment. A team of athletes paid R400 for a day's practice. Let x be the number of athletes attending the training.

   ....

6. Bennie has R54 with which to buy chocolate for his friends. Each chocolate costs R6. How many chocolates can he buy for that amount? Let x be the number of chocolates that Bennie can buy.

   ....

7. Write an equation to calculate the area of a rectangle with length 2,5 cm and breadth 2 cm. Let A represent the area of the rectangle.

   ....

8. There are 38 girls in Grade 7. This is 6 more than double the number of boys.

   ....

9. Janine is 12 years old. Her father's age is 7 years plus three times Janine's age.

   ....

MAKING SENSE OF EQUATIONS

1. Rajbansi Taxi Service charges R10 per kilometre travelled and a standard charge of R30 per trip. Consider the equation below about a taxi trip:
   $10 \times t + 30 = 80$
   (a) Explain what each number and letter symbol stands for in the equation.

   ....

   ....

   ....

   ....

   (b) Why is *t* multiplied by 10 in the equation?

   ....

2. The cost of an adult's ticket for a music concert is four times the cost of a child's ticket. An adult's ticket costs R240. The equation below represents this problem:
   $4 \times x = 240$
   (a) What does *x* represent?

   ....

   (b) Why is *x* multiplied by 4?

   ....

   (c) Solve the equation by inspection.

   ....

   (d) How much does a child's ticket cost?

   ....

3. There are 12 eggs in a carton. Consider the equation below:
   $12 \times c = 72$
   (a) What does the letter symbol *c* represent in the equation?

   ....

   (b) What value of *c* makes the equation true?

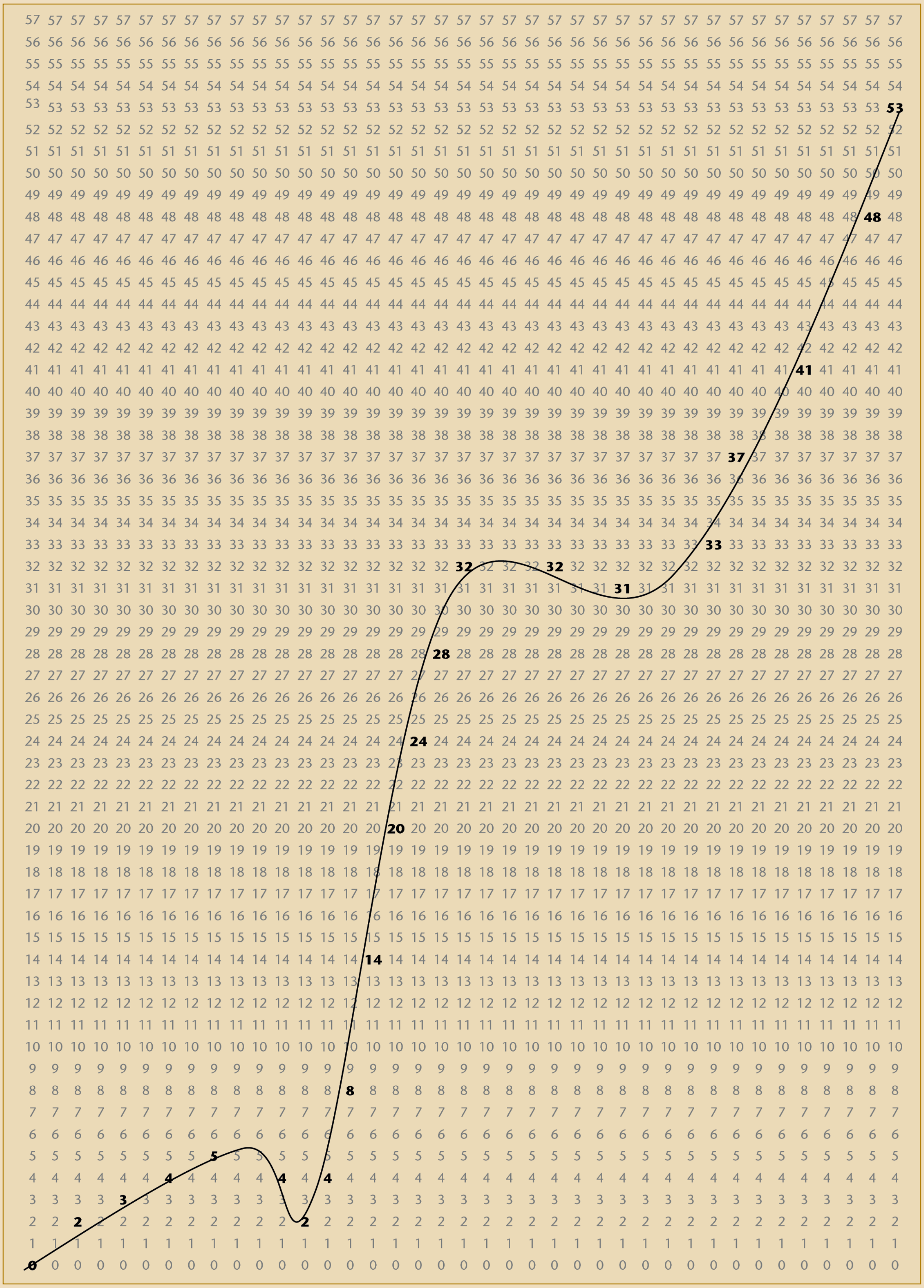
   ....

   (c) What does the number 72 represent?

   ....

Chapter 5
Graphs

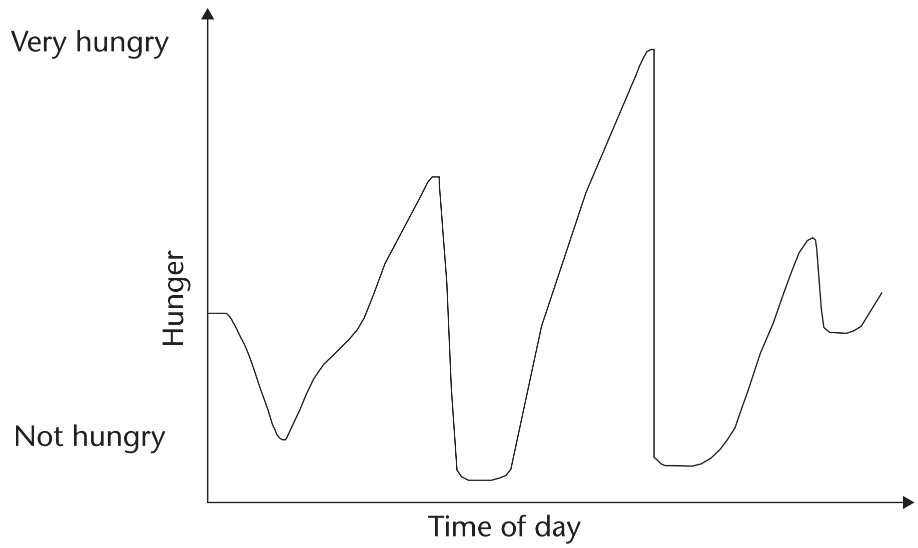
In this chapter, you will get familiar with a new kind of graph: the line graph. A line graph shows how the change in one variable affects another variable. You will specifically deal with global graphs. These graphs show visually how variables vary, focusing on trends rather than detailed readings.



5 Graphs

5.1 A graph can tell a story

1. Jena drew this graph to show how her feelings of hunger changed during the day. Describe in a short paragraph how her day went as far as need for food is concerned.



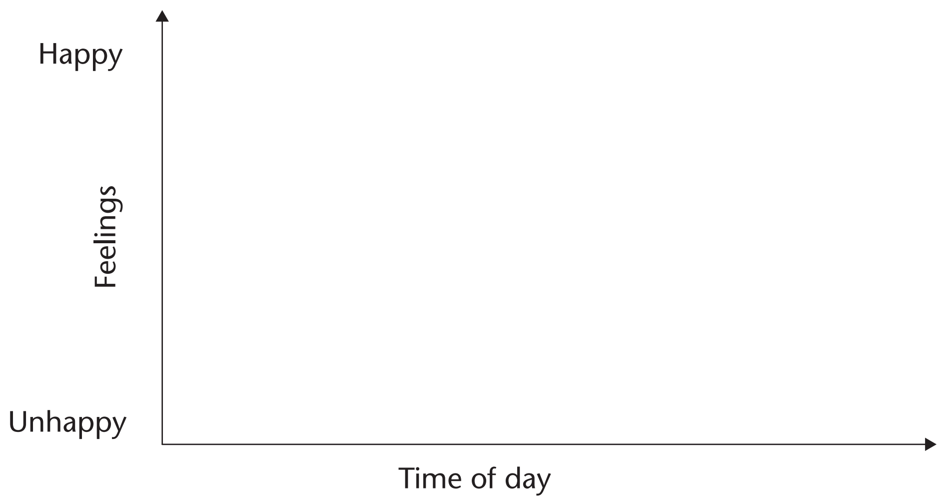
....

....

....

....

2. Think about a specific day and things that happened to you on that day. Draw a graph to show how your feelings changed during that day.



5.2 Investigating rate of change in situations

COMPARE SITUATIONS AND REPRESENT THEM IN A DIFFERENT WAY

1. Consider the situations in (a) and (b) below and complete the tables to represent the relationships.

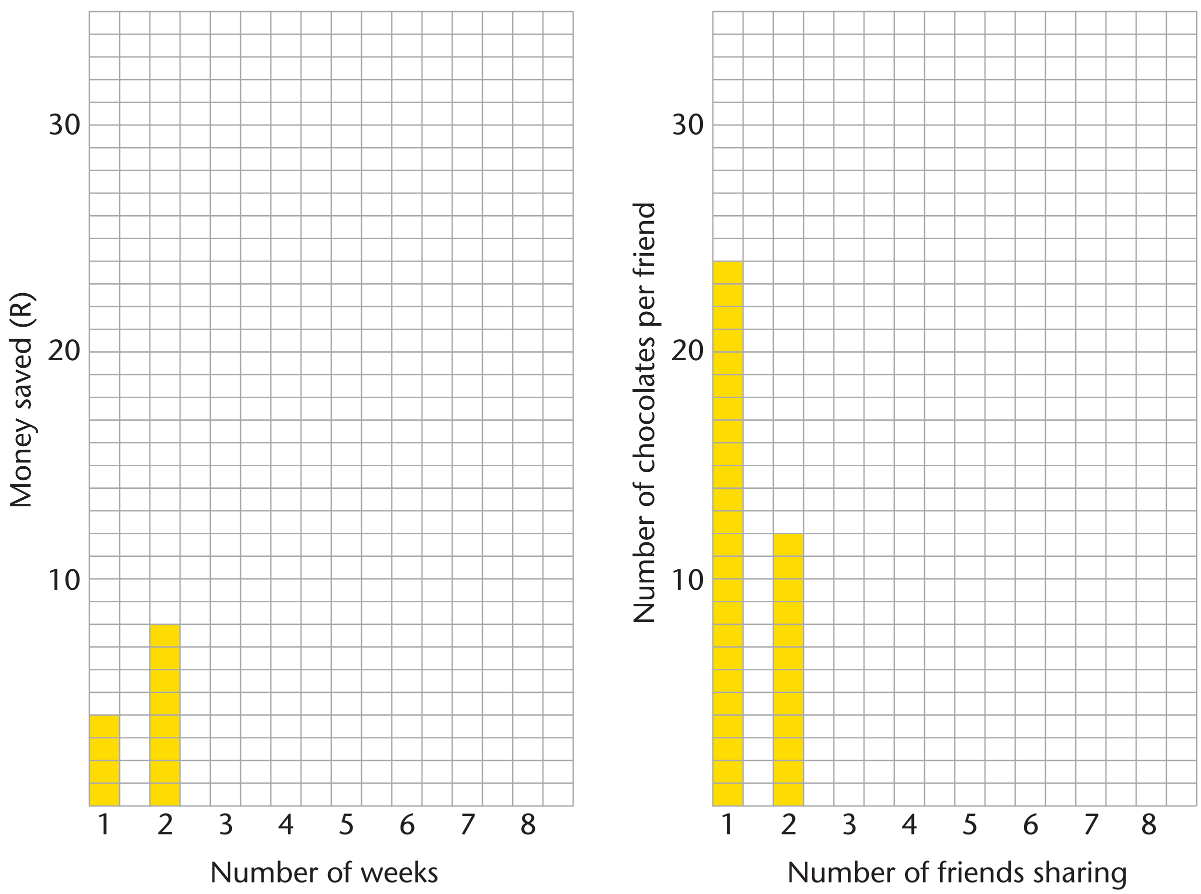
   (a) Sally is saving money to buy a CD that she badly wants. She saves R4 per week.

Number of weeks	1	2	3	4	5	6	7	8
Money saved in rands	4	8						

   (b) Nathi has a box of 24 chocolates. He is thinking about sharing the chocolates equally between different numbers of friends and is working out how many chocolates each friend would get.

Number of friends	1	2	3	4	5	6	7	8
Chocolates per friend	24	12						

   (c) On the grids below, draw bar graphs to represent the relationships in the situations described in (a) and (b). The length of each bar should represent an output number.



2. Consider the situations in (a) and (b) below and complete the tables to represent the relationships.

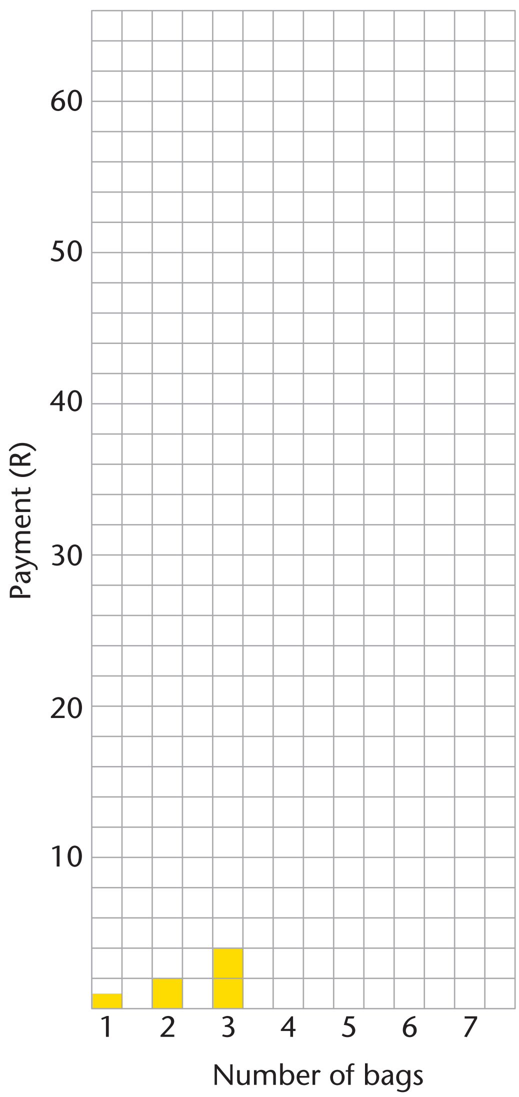
(a) Vusi collects acorns for a pig farmer who pays him per bag. Vusi thinks: "I wish Mr Bengu would agree to pay me R1 for the first bag of acorns, R2 for the second bag, R4 for the third bag and then keep on doubling the money for every bag."

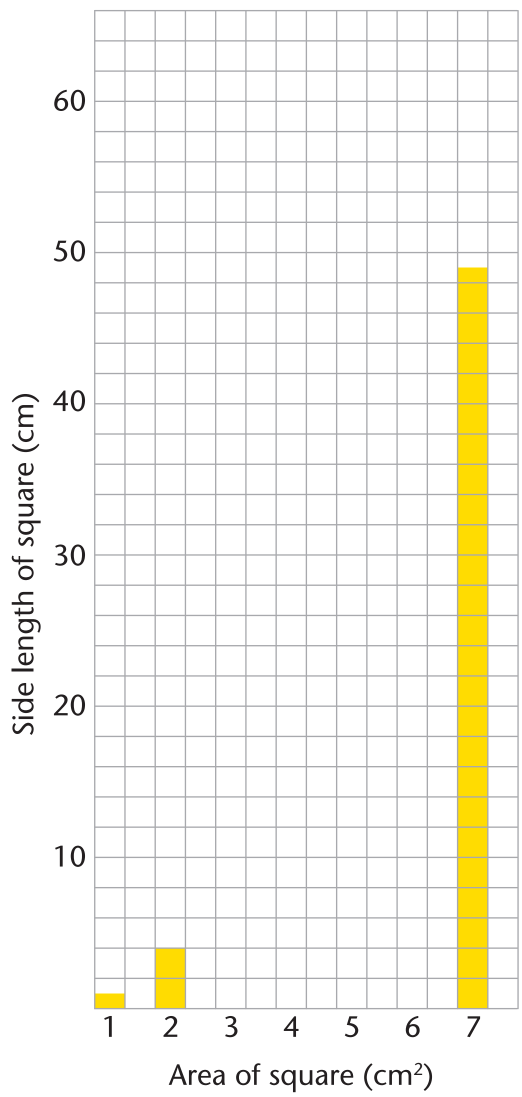
Number of bags	1	2	3	4	5	6	7
Payment (R)	1	2	4	8			

(b) Judy works out the areas of squares with different side lengths.

Side length of square (cm)	1	2	3	4	5	6	7
Area of square (cm^2)	1	4					

(c) On the grids below, draw bar graphs to represent the relationships in the situations described in (a) and (b). The length of each bar should represent an output number.





3. The input numbers for the different relationships in questions 1 and 2 are the same, but the output numbers differ. Describe how the output numbers change in each of the four situations.

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4. Describe briefly how the shape of the bar graphs differ.

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5. Turn back to the tables of values that you made for the four relationships in questions 1 and 2. Find out how the output values changed by calculating the differences between consecutive output values:

(a) Sally's savings: 4 8 12 16

Differences: 4 4

Sally's savings grow by .... every week.

(b) Chocolates per friend: 24 12 8

Differences: 12

The number of chocolates per friend ....

....

(c) Vusi's payment per bag: 1 2

Differences: 1

The amount grows slowly at first and then ....

(d) Areas of Judy's squares: 1 4 9

Differences: 3

The area of the squares ....

The amount of money that Sally saves per week stays **constant**. The relationship therefore has a **constant rate of change**.

With every additional friend, the number of chocolates per friend changes. The number of chocolates changes (decreases) **rapidly at first and then it changes slower**. The **rate of change is not constant**.

The amount that Vusi would like to earn per bag of acorns grows faster and faster with each bag that he collects. The **rate of change increases**.

The area of a square also **increases faster and faster** for every cm added to the side length.

The **rate of change** means how fast or slow change happens per unit of time.

The **shape** of a bar graph shows the **rate of change** of the relationship. If the rate of change is constant, the shape is a **straight line**. If it is changing, the shape is a **curve**.

6. Refer to the bar graphs in questions 1 and 2 and link the shape of the graphs to the rate of change of the relationship.

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5.3 Interpreting graphs

READING GRAPHS

1. Look carefully at the graph.

(a) What does this graph tell you?

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(b) Explain your answer in (a).

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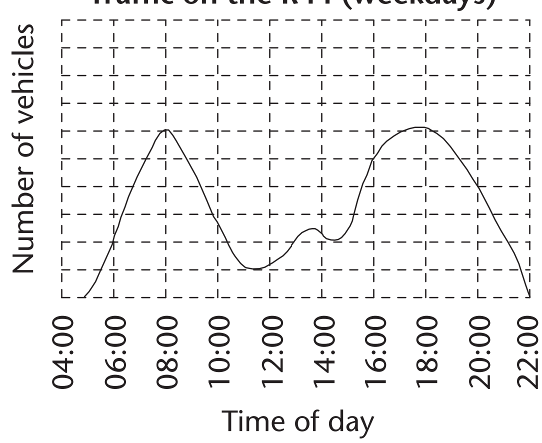
Traffic on the R44 (weekdays)



2. Mr Thatcher bought three plants in containers. The salesman at the nursery told him that one of the plants, Glamiolus, grows at a constant rate. The second plant, Bouncy Bess, grows slowly at first but then grows faster and faster. The salesman was not sure about the rate at which the third plant, Samara, grows.

(a) What does "grows at a constant rate" mean?

....

....

Mr Thatcher measured the three plants every week and recorded the heights in a table, given below.

(b) Calculate the differences in height from week to week, to find the rate at which each plant grows per week.

Week	Height of A (cm)	Height of B (cm)	Height of C (cm)
1	6	8,3	10,1
2	6,3	10,2	10,6
3	6,4	12,2	11,2
4	7,2	14,1	11,9
5	7,3	16,2	12,8
6	7,4	18,3	13,9
7	9,1	20,2	15,8

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(c) Identify the plants. Which plant is plant A, which plant is plant B and which plant is plant C? Explain how you got your answers.

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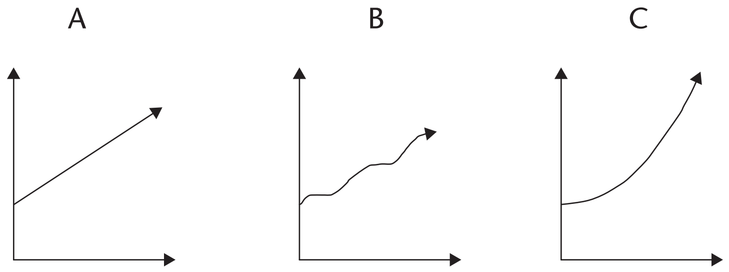
(d) The three graphs below show the growth of the three plants. Which graph belongs to which plant? Explain.



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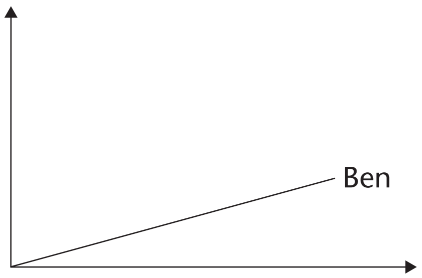
REVISITING CHANGE AND RATE OF CHANGE

In section 5.2 you compared the way in which relationships changed.

1. Consider the following situations:
   Ben saves R5 per week. Sally saves R7 per week.
   Charlie saves R5 in the first week, R6 in the second week and R7 in the third week. Every week he increases the amount that he saves by R1.

   (a) The graph shows Ben's savings. Draw Sally's savings and Charlie's savings. Do it on the same sketch.



   (b) Describe and explain the shape of the graphs showing Sally, Ben and Charlie's savings.

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The rate of change in a relationship influences the **steepness** of the graph. The higher the rate of change (i.e. the faster the output numbers change), the steeper the graph.

2. Examine the following relationships. Complete the tables and calculate the differences between the output numbers indicated with arrows.

(a) Christine wants to buy a book for her favourite teacher. The book costs R240. This is a lot of money for Christine to spend. She realises that if she asks her friend Beatrice to share the cost, she will have to spend only R120. She could even ask more classmates to join in and share the cost. Christine investigates the situation and calculates what amount everyone must pay if they share the cost equally.

Number of learners sharing the cost	1	2	3	4	6	8	10	12
Amount each learner will pay	240	120						

120

(b) Investigate the relationship between the length of a side of a square and the perimeter of the square.

Length of a side of the square	1	2	3	4	6	7	8	9
Perimeter of the square	4	8	12					

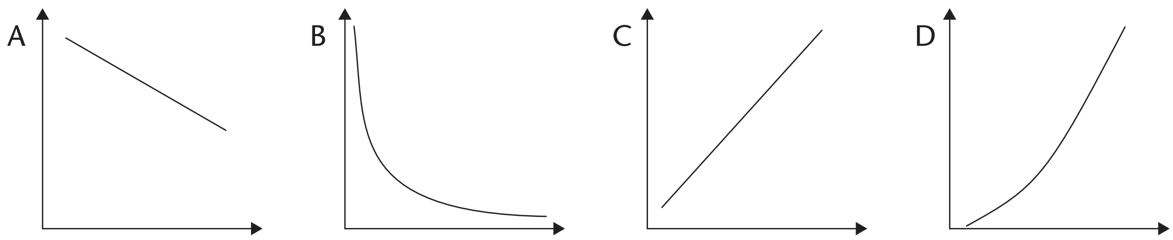
(c) Investigate the relationship between the length of a side of a square and the area of the square.

Length of a side of the square	1	2	3	4	6	7	8	9
Area of the square	1	4	9					

(d) A tall candle was lit and its length was measured and recorded every hour while it was burning.

Number of hours the candle is burning	1	2	3	4	6	7	8	9
Length of candle in cm	33	31	29					

3. Match each of the graphs below to one of the situations in question 2.



Write the letter of the graph next to the description of the situation:

(a) Buying a book for the teacher ....

(b) Length of a side of a square and the perimeter of the square ....

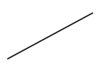
(c) Length of a side of a square and the area of the square ....

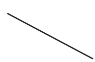
(d) Length of the candle and the number of hours it is burning ....

When we investigate the growth (or change) in a relationship, we look at the way the output numbers change.

The change can be:

- an increase or a decrease
- a constant increase, for example the perimeter of a square as the side length increases
- a constant decrease, for example the length of a burning candle
- an increase that is not constant but happens faster and faster, for example the area of a square as the side length increases
- a decrease that is not constant but happens faster and faster, for example the amount of money each friend has to pay as more and more friends share the cost.

In the case of an increase, the graph slopes like this:  or  or 

In the case of a decrease, the graph slopes like this:  or  or 

When the increase or decrease is constant, the graph is a straight line and it is called a **linear graph**.

If the increase or decrease is not constant, the graph is curved and is called a **non-linear graph**.

If there is no change in the output variable, the graph is a straight horizontal line.

4. Consider the graphs in question 3 on the previous page.
   (a) Which graphs indicate a linear increase or decrease?

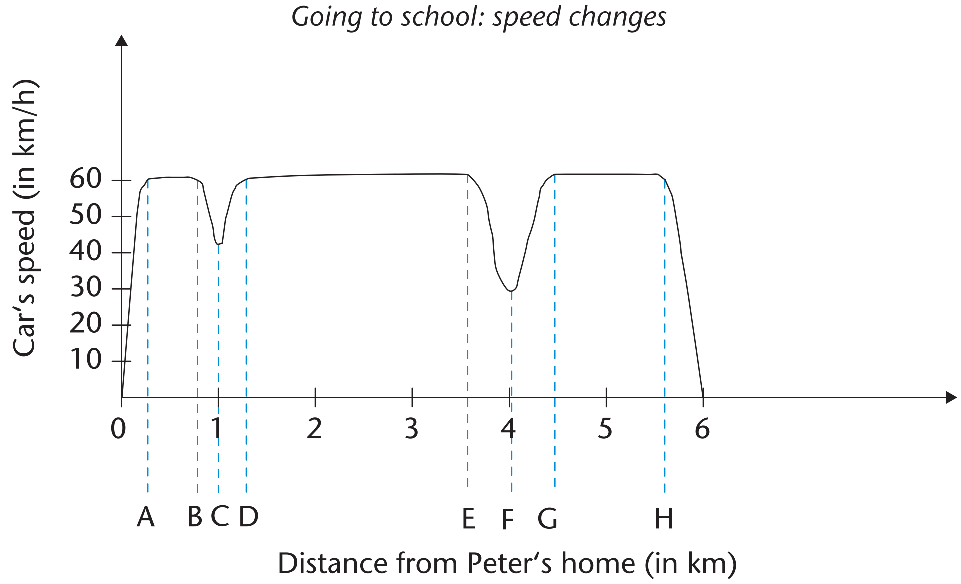
   ....

   (b) Which graphs indicate a decrease or increase which is not constant?

   ....

5. Peter's father drives him to school in the mornings. Below is a graph of their journey to school. Describe the story that the graph tells. What do you know about the route that they are taking?

Going to school: speed changes



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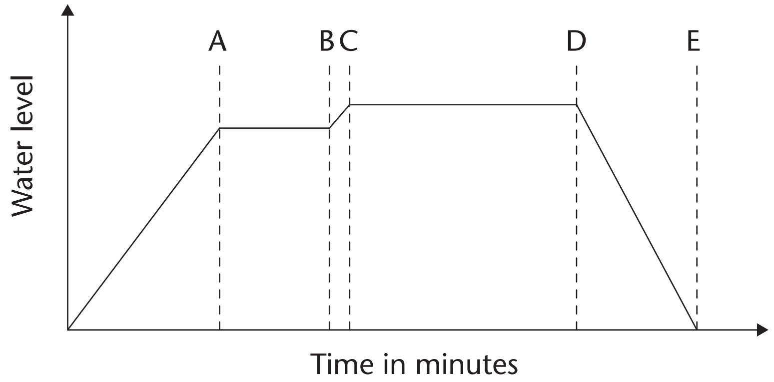
6. Consider the graph in question 5 above. Identify the parts of the graph that are increasing, decreasing and constant.

0 to A:	A to B:	B to C: ....
C to D:	D to E:	E to F: ....
F to G:	G to H:	H to 6: ....

EXPLORING MORE GRAPHS

1. Janet takes a bath. The graph below shows the height of the water level in the bathtub as time passes. The water runs into the bath at a constant rate. Study the graph and describe what happens.



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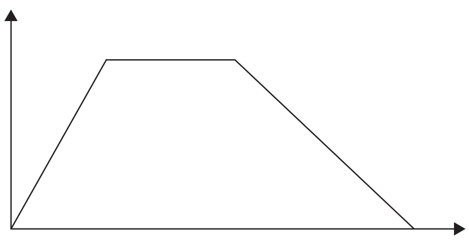
2. The axes of the graph below are not labelled.



The **vertical axis** is the one that goes from bottom to top. The **horizontal axis** is the one that goes from left to right. (**Axes** is the plural of **axis**.)

(a) Which of the following sets of labels could fit the graph?

A: **vertical axis:** time passed; **horizontal axis:** distance from home

B: **vertical axis:** distance from home; **horizontal axis:** time passed

C: **vertical axis:** rainfall; **horizontal axis:** temperature

....

(b) Describe the story told by the graph, with the axes that you chose.

....

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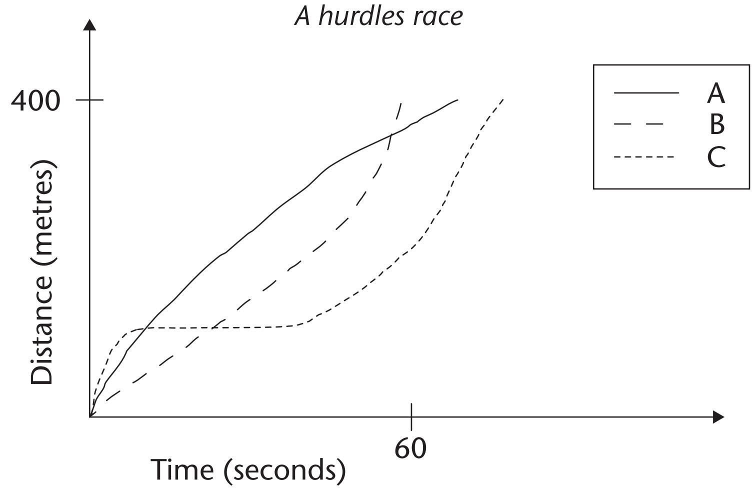
3. The graph below shows the distance that three athletes, A, B and C, covered in a hurdles race in a certain time.
   (a) Describe what happened during the race.



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   (b) How far was the race? ....

   (c) Which of the athletes, A, B or C, won the race? ....

   (d) Did the best athlete of the three win? Explain your answer.

....

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....

4. Identify the graphs (or parts of a graph) in questions 1, 2 and 3 above that are linear and those that are non-linear.

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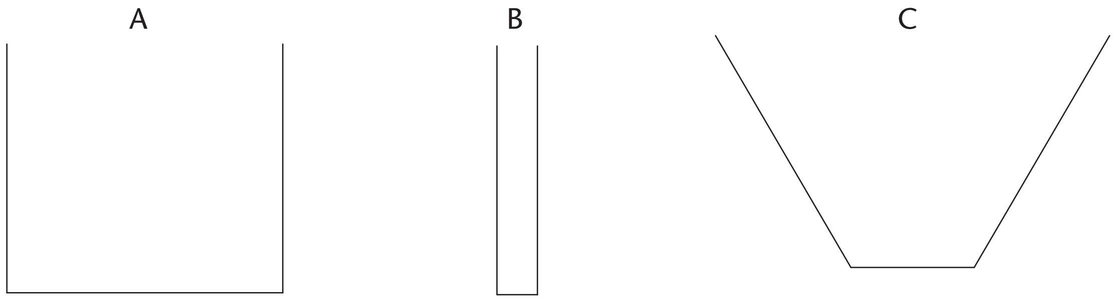
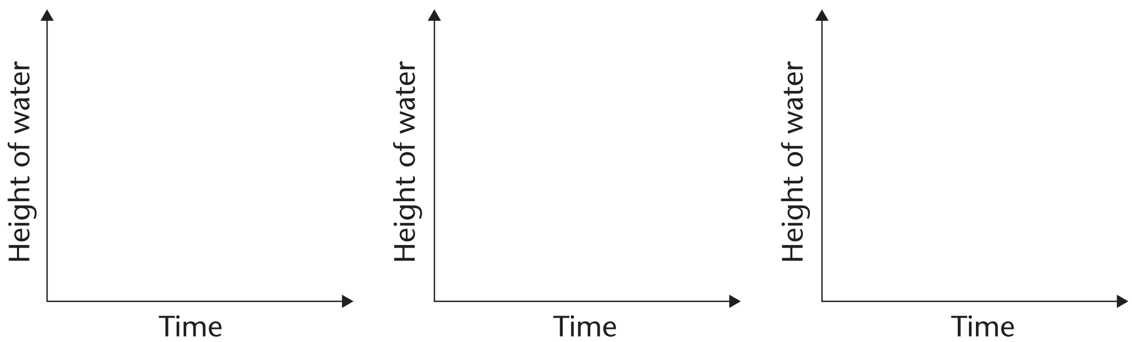
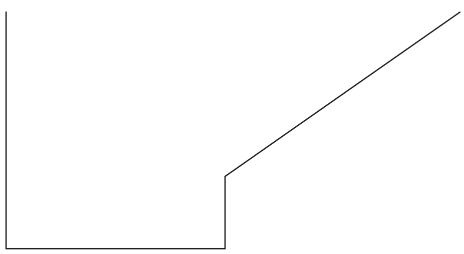
5.4 Drawing graphs

1. Water is dripping at a constant rate into three containers, A, B and C, shown below. Draw graphs to show how the height of the water in each container will vary with time.



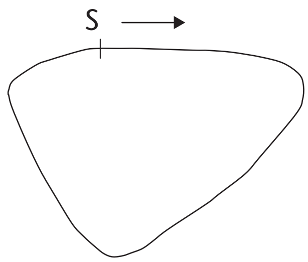


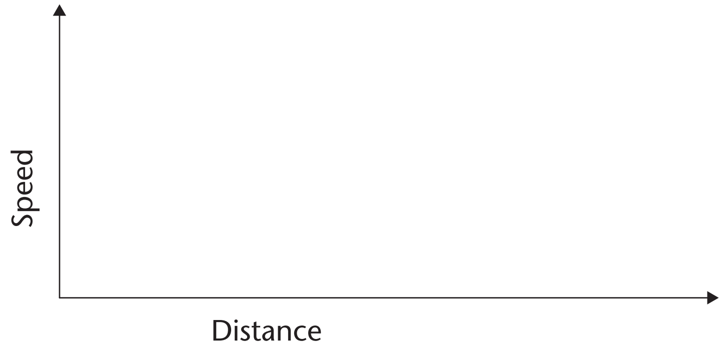
2. Draw a graph showing the height of the water level in the swimming pool below if the pool is filled with a constant stream of water.



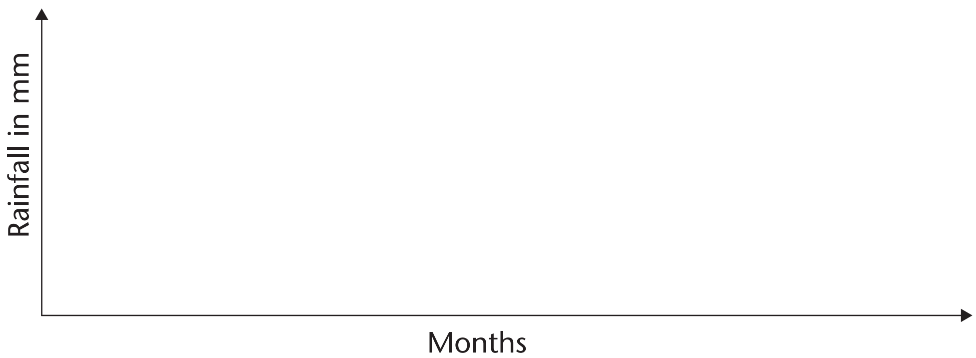
Cross-section of swimming pool

3. Draw a graph of the speed of a racing car as it travels once around the track shown below. S is the starting point.





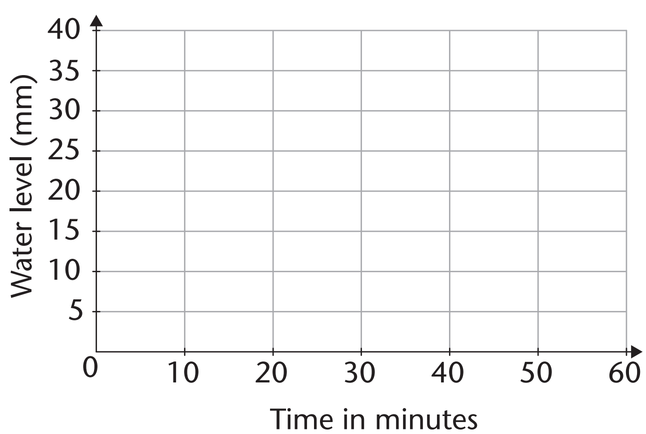
4. The Western Cape gets rain during the winter months, but in summer it is usually dry. Draw a global graph of the average rainfall in the Western Cape during one year.



5. Draw a graph of the following story:

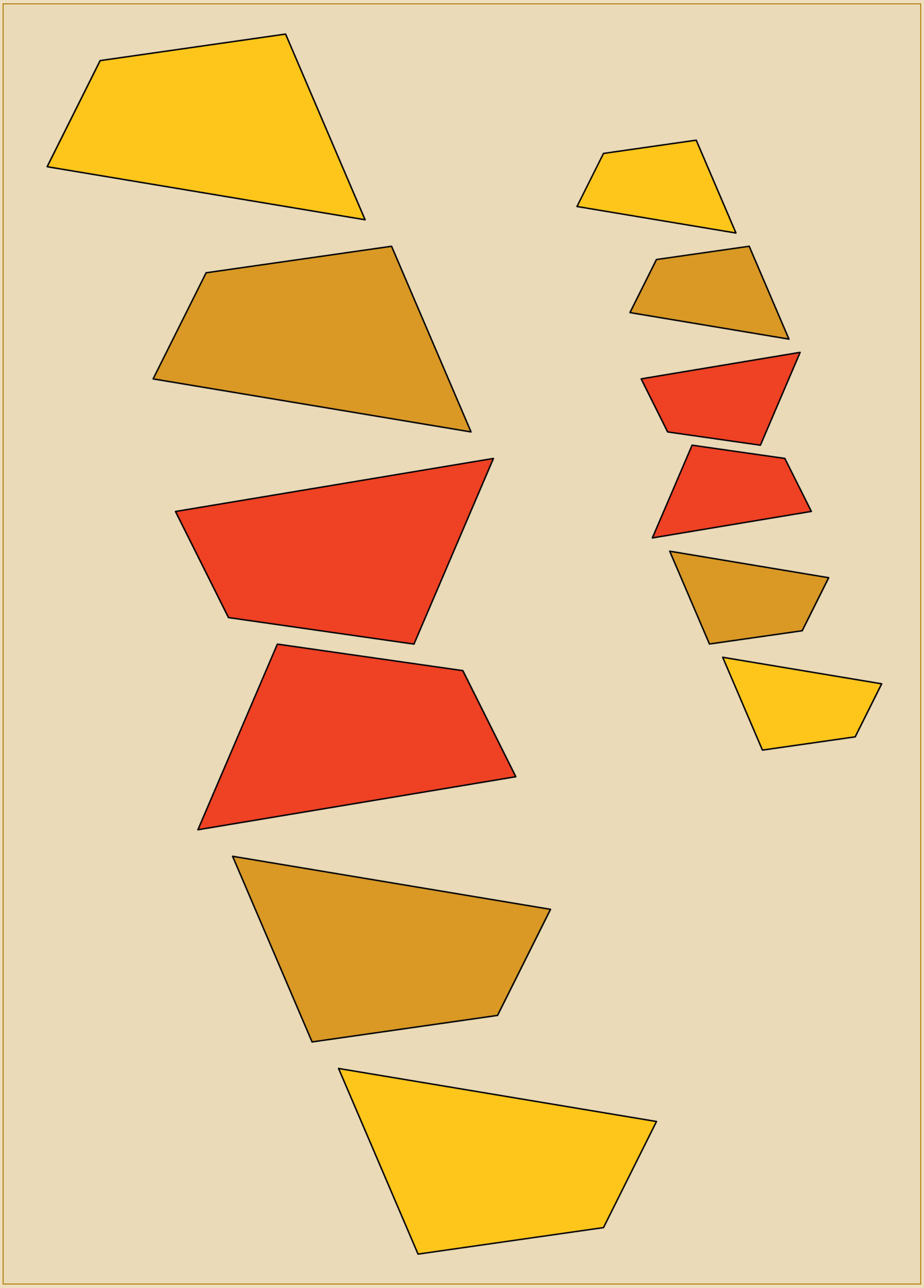
   During a rainstorm, Lydia put a measuring cup outside to measure the rainfall. After 10 minutes of hard rain the water level was 10 mm. It started to rain softer, and after 20 more minutes the water level was 15 mm.

   When Lydia went back 10 minutes later, the level was 30 mm. An hour after the storm started, the water level was still 30 mm.



Chapter 6
Transformation geometry

In this chapter, you will revise the property of symmetry and practise identifying lines of symmetry in geometric figures. You will then investigate how figures can be reflected, rotated or translated, while the size and shape of the original figure remains the same. You will also investigate how we can change the size of a figure, but still keep the angles of the figure the same, to produce enlarged or reduced similar figures. In such figures, you will work out the factor by which the original figure was resized.



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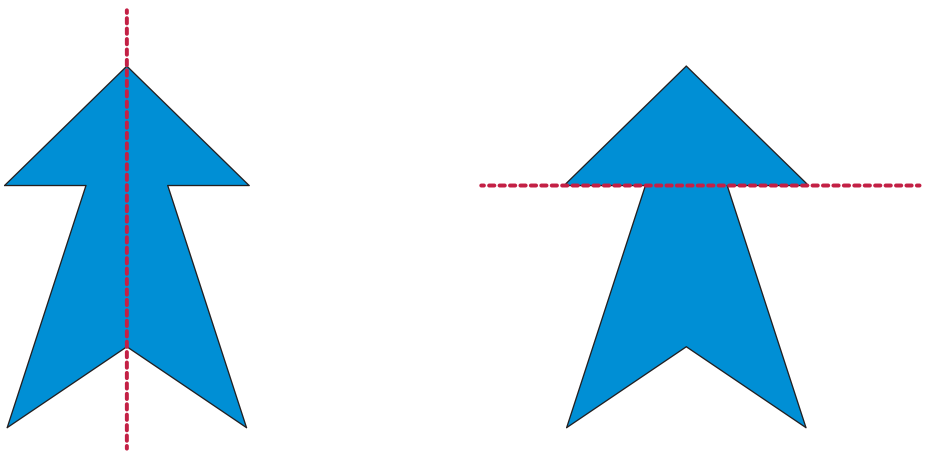
6 Transformation geometry

6.1 Lines of symmetry

WHAT IS THE LINE OF SYMMETRY?

In the diagrams below, the red dotted lines divide the arrows into two parts. In which diagram does the red dotted line divide the arrow into two parts that are exactly the same?

....



Arrow A *Arrow B*

If you were to cut out arrow A and fold it along the red dotted line, the two parts would fit perfectly on top of one another (all edges would match). The fold line is called a **line of symmetry** or an **axis of symmetry**.

A line or axis of symmetry is a line that divides a figure into two parts that have an equal number of sides, and all the corresponding sides and angles are equal. The two parts on either side of the line of symmetry are mirror images of each other. We also say the parts are **congruent**.

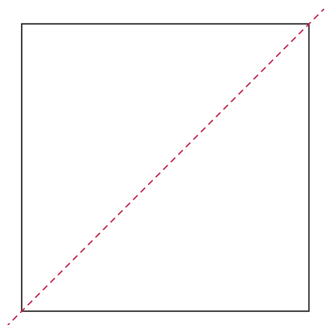
A geometric figure can have no line of symmetry, one line of symmetry, or more than one line of symmetry.

Congruent figures are figures that are the same size and shape. All the sides and angles of the figures match.

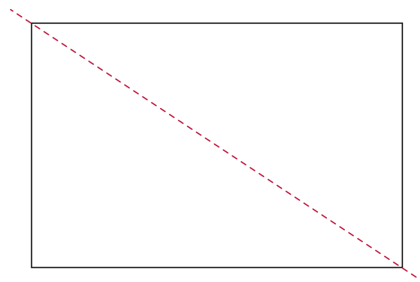
IDENTIFYING LINES OF SYMMETRY

1. (a) Make a tick next to each figure in which the red line is a line of symmetry.
   (b) In the figures where the red line is not a line of symmetry, draw in a line of symmetry if this is possible. If there is more than one line of symmetry, draw it in too. If a figure doesn't have any lines of symmetry, write this above the figure.

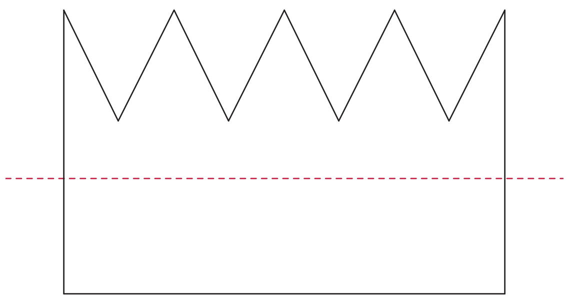
A



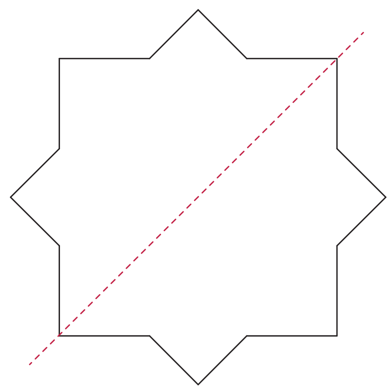
B



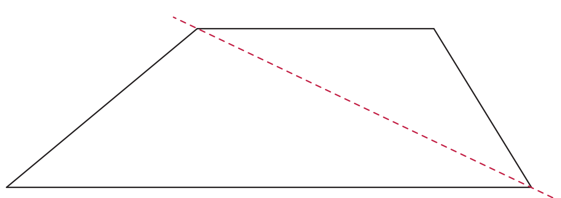
C



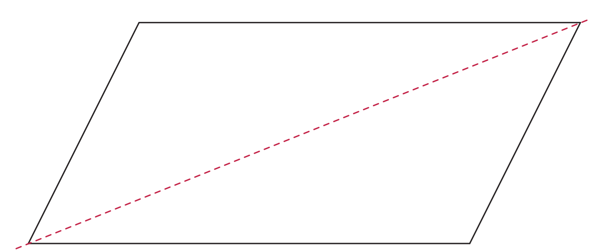
D



E

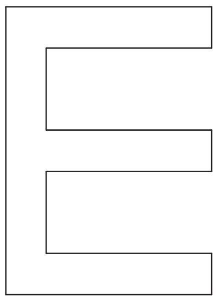


F

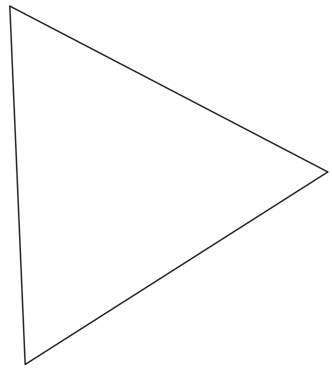


2. Draw lines of symmetry in the following geometric figures. Also write down how many lines of symmetry there are in each figure.

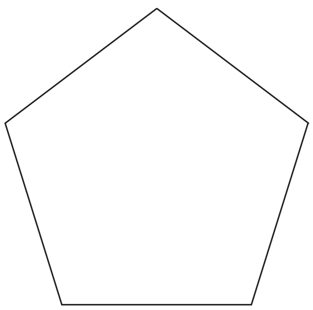
A



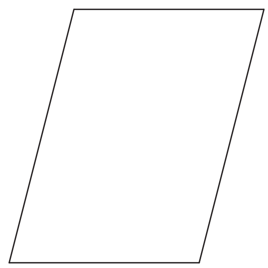
B



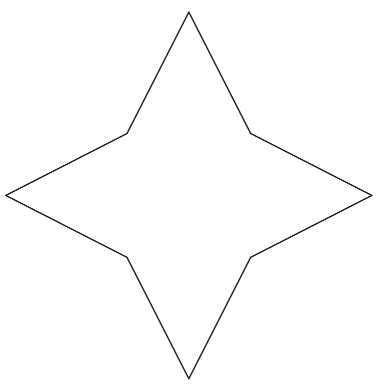
C



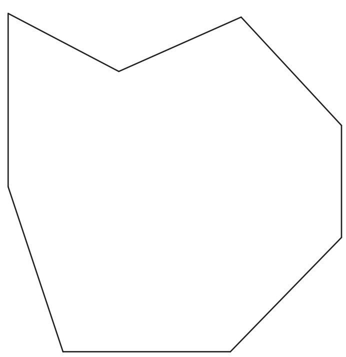
D



E

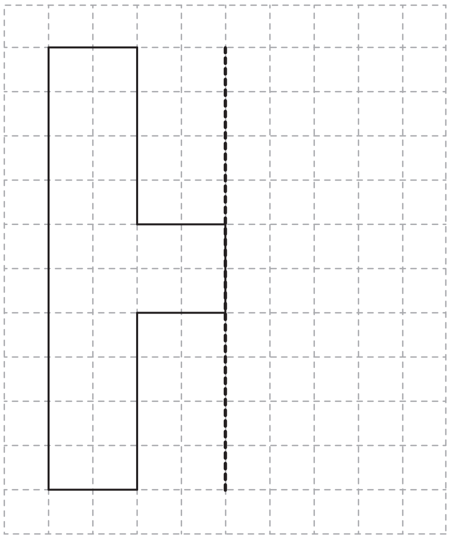


F

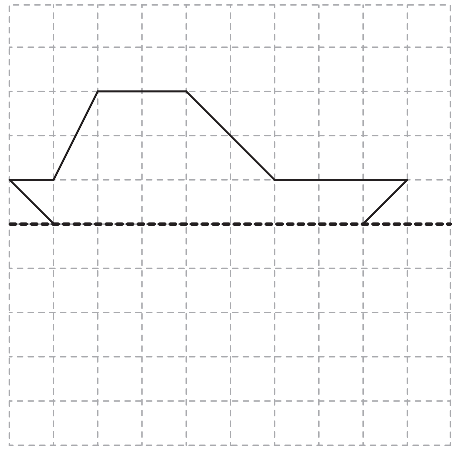


3. In each diagram, the dotted line is the axis of symmetry. Complete each figure.

A



B



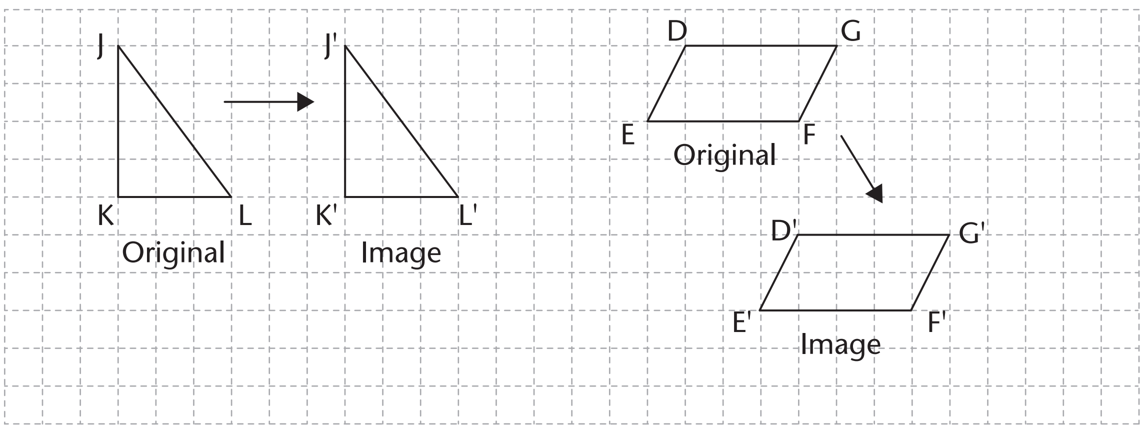
6.2 Original figures and their images

Figures can be moved around in different ways – they can be shifted, swung around and turned over. When the movement is done, the figure in its new position is called the **image** of the original figure.

> Figures can be moved in three ways: through **translation**, **reflection** and **rotation**. These transformations are often referred to as "sliding" (shifting), "flipping" (turning over) and "turning" (swinging) respectively.

6.3 Translating figures

Here are two original figures and their images after the figures were **translated**:

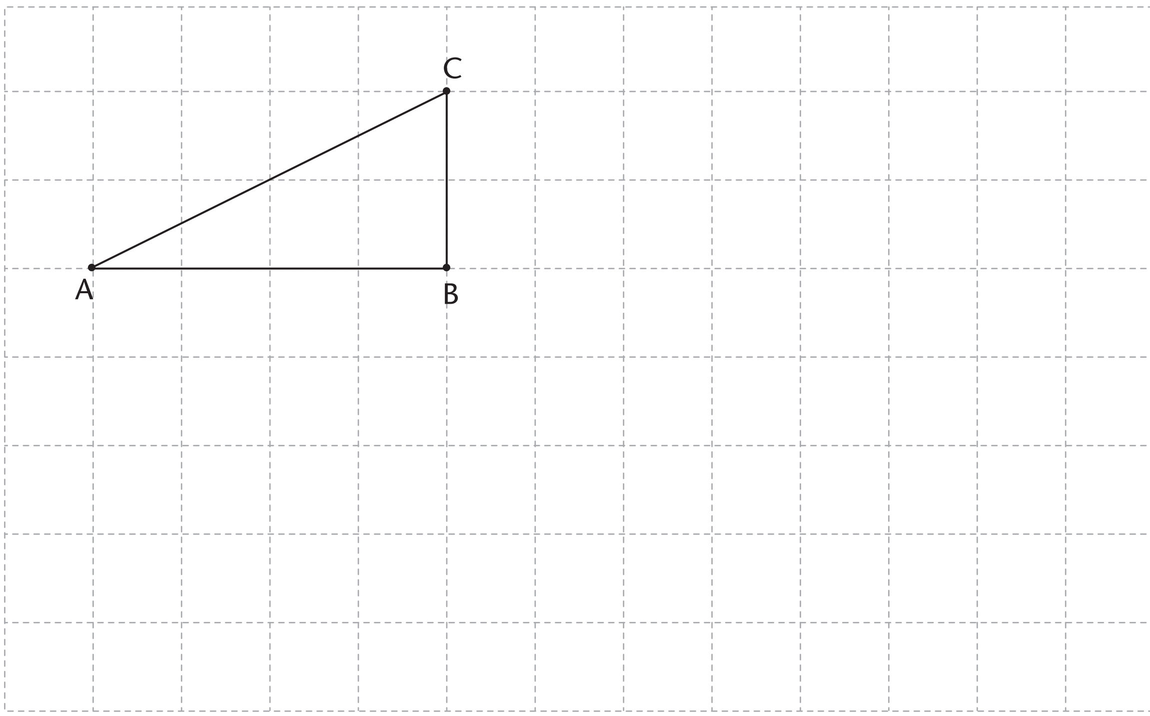


When we name the image, we use the same letters for the points that correspond to those of the original figure, but we add the prime symbol (') after each letter. The image of ΔJKL is ΔJ'K'L'. The image of parallelogram DEFG is parallelogram D'E'F'G'.

INVESTIGATING THE PROPERTIES OF TRANSLATION

In a **translation**, all the points on the figure move in the same direction by the same distance. For example, look at ΔJKL on the previous page. All of its points have moved 6 units to the right. Also look at parallelogram DEFG on the previous page. All of its points have moved 3 units to the right and 5 units down.

1. Look at ΔABC below.
   (a) Translate each of the points A, B and C 5 units to the right and 2 units down. Then join the translated points to form the image ΔA'B'C'.



Look at the completed translation.

(b) Are the side lengths of the original triangle and those of its image the same?

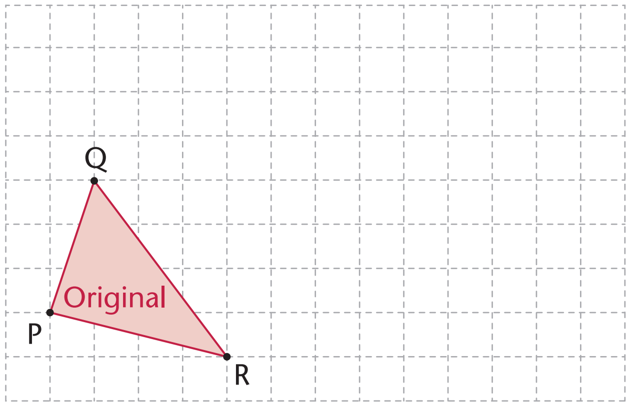
....

(c) Is the area of the original triangle the same as the area of its image?

....

2. Look at ΔPQR below.

(a) Translate each of the points P, Q and R 4 units to the right and 2 units up. Then join the translated points to form the image ΔP'Q'R'.



(b) Join point P and its image, point Q and its image, and point R and its image.

(c) Are the line segments that join the original points to their image points equal in length?

....

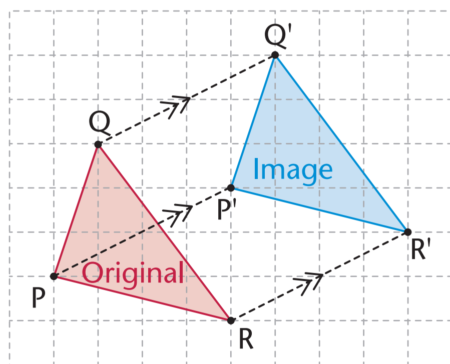
(d) Are the line segments that join the original points to their image points parallel?

....

Properties of translation

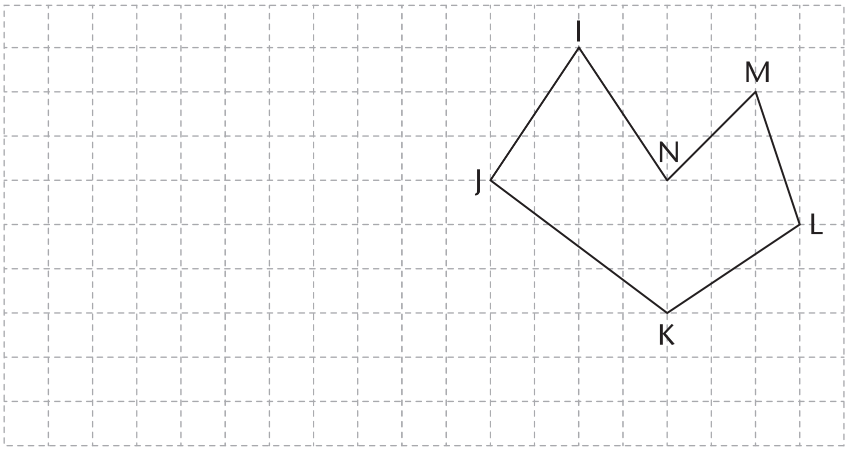
Use the diagram on the right to check if the following is true:

- The line segments that connect the vertices of the original figure to those of the image are all equal in length:
  PP' = RR' = QQ'
- The line segments that connect the vertices of the original figure to those of the image are all parallel to one another:
  PP' || RR' || QQ'
- When a figure is translated, its shape and size do not change. The original and its image are therefore congruent.

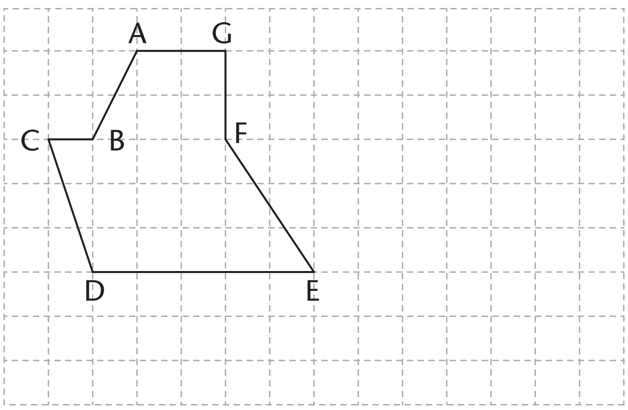


PRACTISE TRANSLATING FIGURES

1. Translate the following figure 8 units to the left and 2 units down.

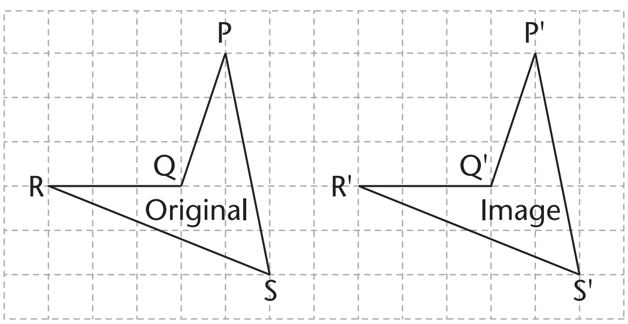


2. Translate the following figure 6 units to the right and 1 unit down.

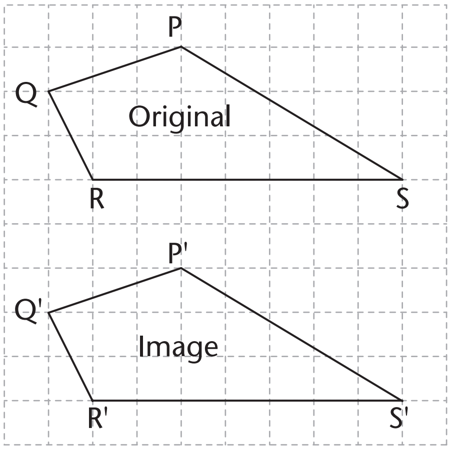


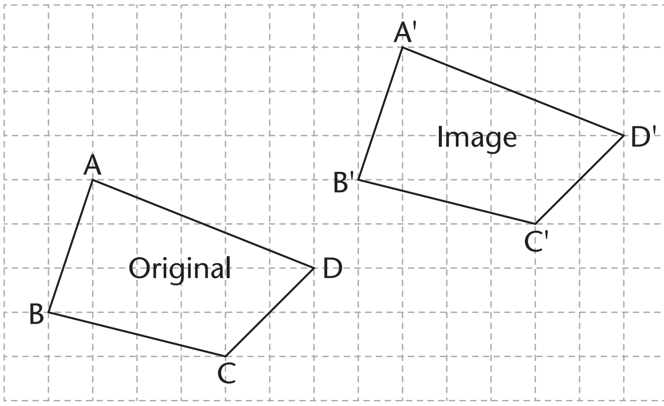
3. Describe the translation in each of the following diagrams:

   (a) ....



(b) .... (c) ....





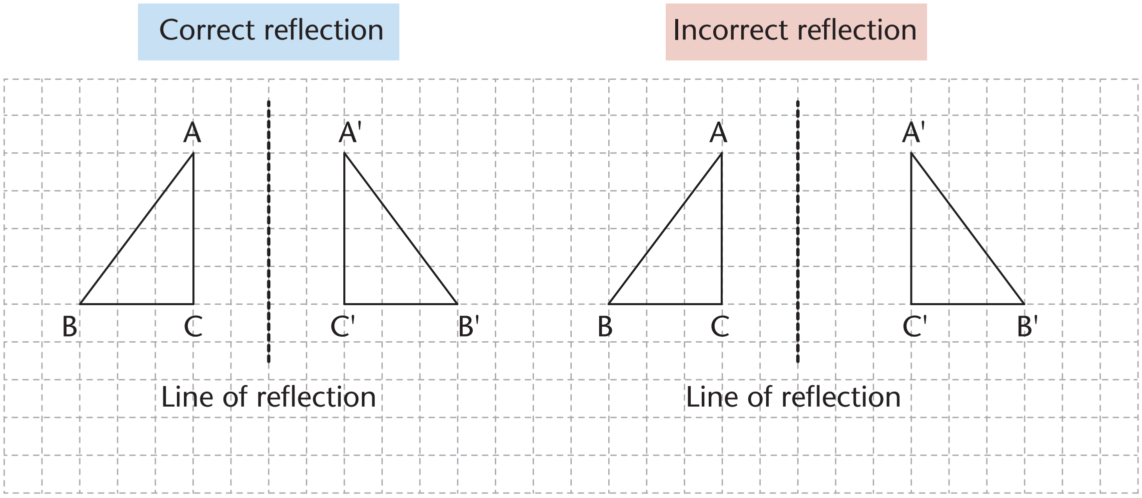
6.4 Reflecting figures

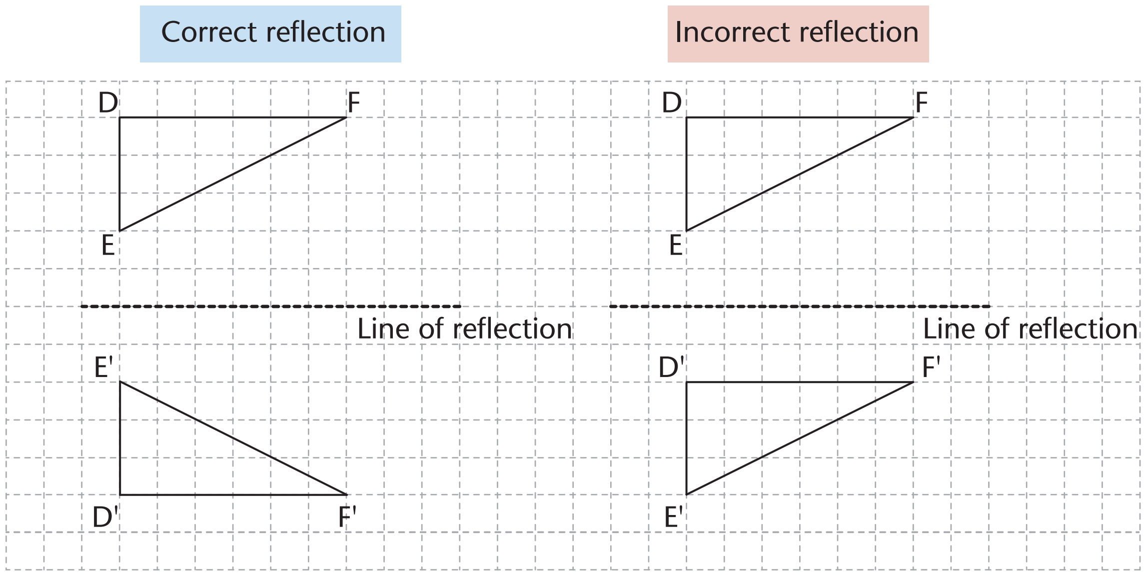
When a figure is **reflected**, it is flipped or turned over. The image that is produced is the mirror image of the original figure. The **line of reflection** is like a mirror in which the original figure is reflected.

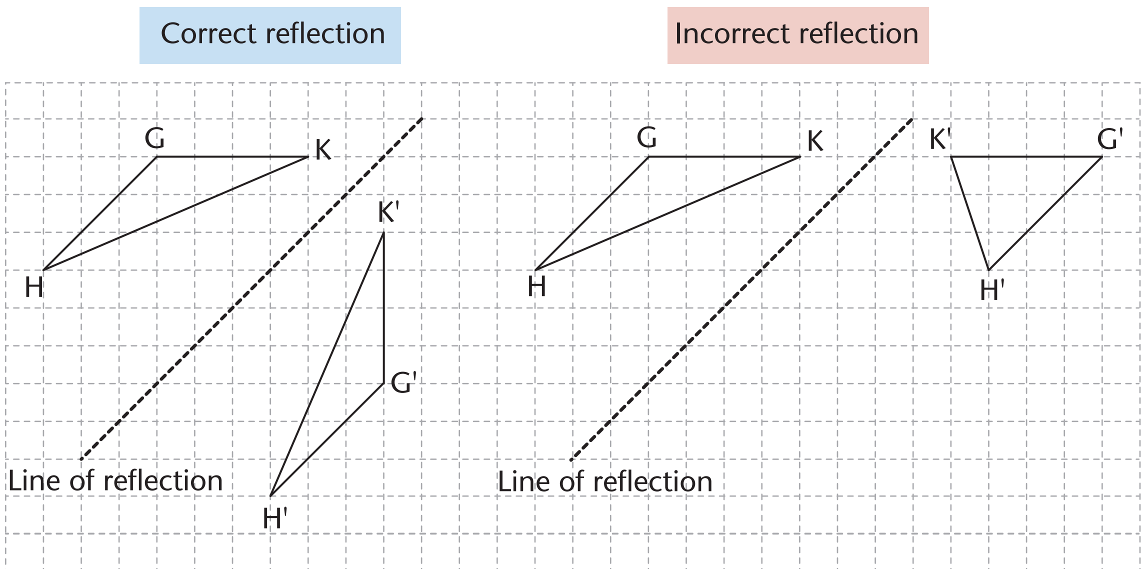
The image is produced on the opposite side of the line of reflection. Each point on the original figure and its corresponding point on the image are the same distance away from the line of reflection.

INVESTIGATING THE PROPERTIES OF REFLECTION

The diagrams below and on the next page show examples of figures that have been correctly and incorrectly reflected in the lines of reflection.







1. Write down the distance from each of the following points to the line of reflection.

Original figure	**Correct reflection**	**Incorrect reflection**
A: 2 units	A':	A':
B:	B':	B':
C:	C':	C':
D:	D':	D':
E:	E':	E':
F:	F':	F':
G:	G':	G':
H:	H':	H':
K:	K':	K':

2. Look at each set of *correct* reflections.
   (a) Are the side lengths of the image the same as those of the original figure?

   ....

   (b) Are the size and shape of the image the same as the size and shape of the original figure?

   ....

3. (a) In each diagram showing the *correct* reflection, draw a dotted line to join each point on the original figure to its corresponding reflected point (A to A', B to B', C to C' and so on).
   (b) Is the line that joins the original point to its correct reflection perpendicular to the line of reflection?

   ....

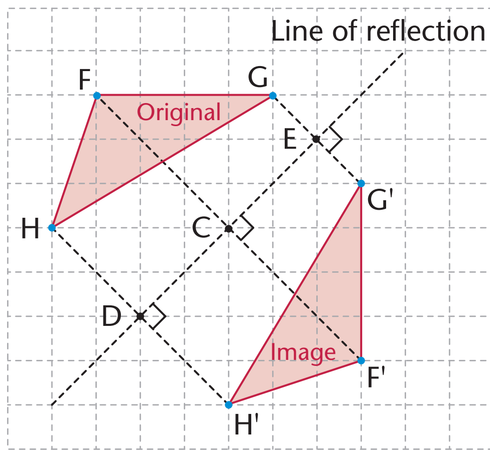
4. (a) In each diagram showing the *incorrect* reflection, draw a dotted line to join each point on the original figure to its corresponding reflected point.
   (b) Is the line that joins the original point to its incorrect reflection perpendicular to the line of reflection?

   ....

Properties of reflection

The diagram on the right shows ΔFHG and its reflection ΔF'H'G'. Notice the following properties of reflection:

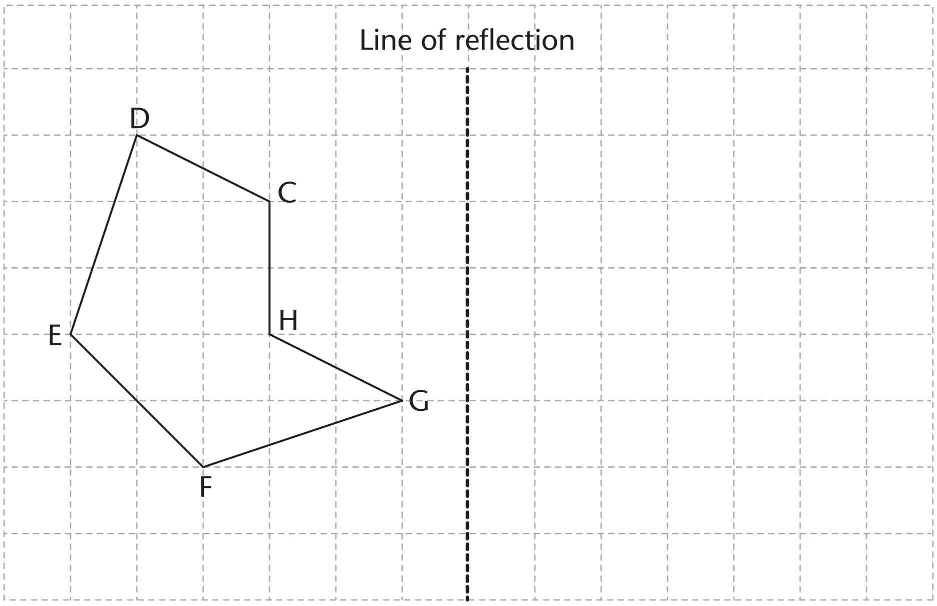
- The image of ΔFHG lies on the opposite side of the line of reflection.
- The distance from the original point to the line of reflection is the same as the distance from the reflected point to the line of reflection: GE = G'E; FC = F'C and HD = H'D
- The line that connects an original point to its image is always perpendicular (⊥) to the line of reflection: HH' ⊥ line of reflection; FF' ⊥ line of reflection, and GG' ⊥ line of reflection.
- When a figure is reflected, its shape and size do not change. The original and its image are therefore congruent.



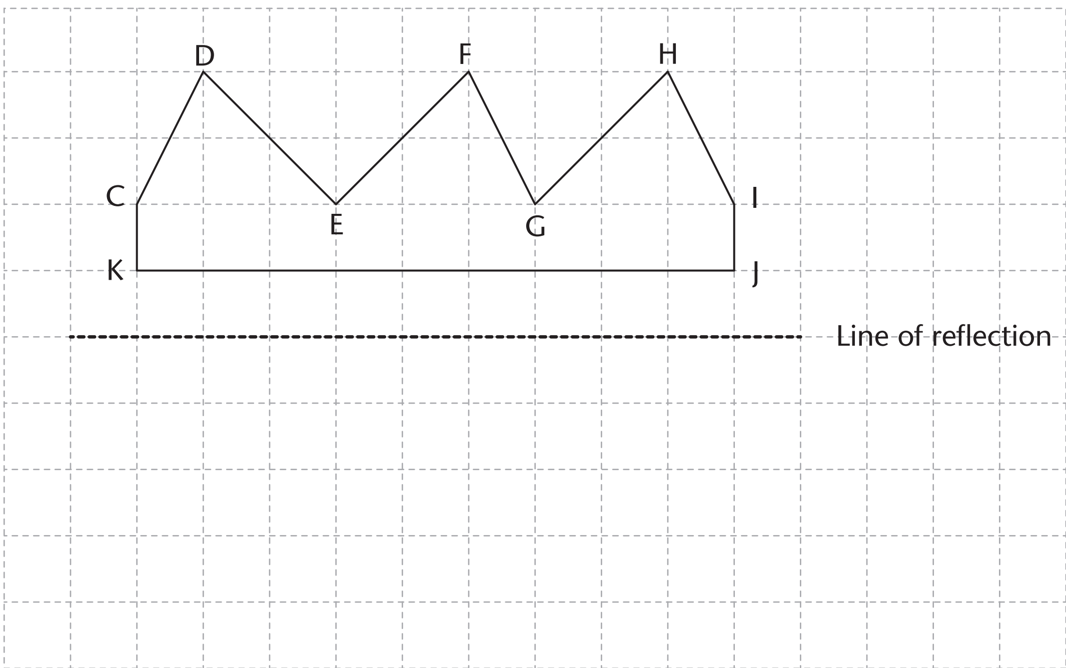
PRACTISE REFLECTING FIGURES

1. Reflect the following figures in the given line of reflection. (*Hint*: First reflect the points; then join the reflected points.)

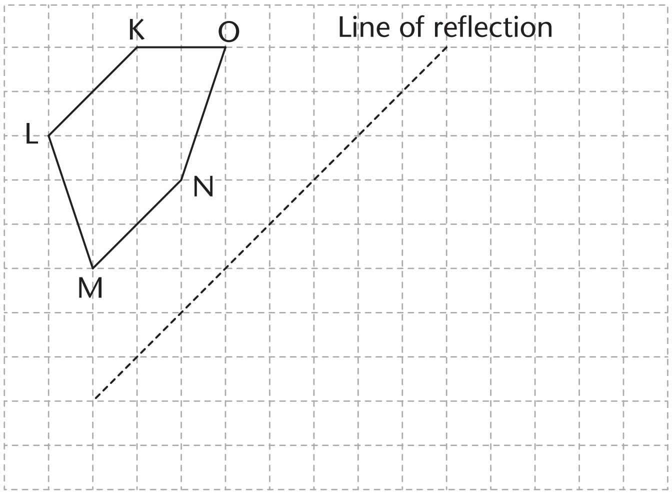
(a)



(b)

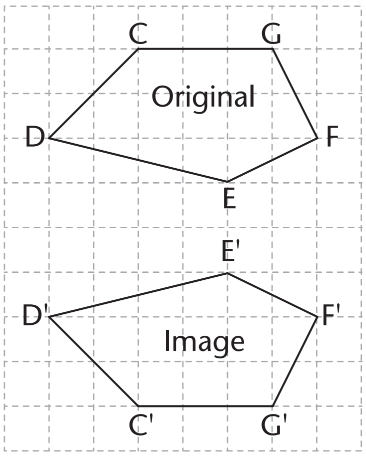


(c)

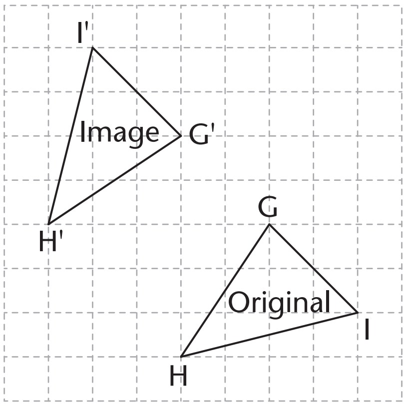


2. Draw the line of reflection.

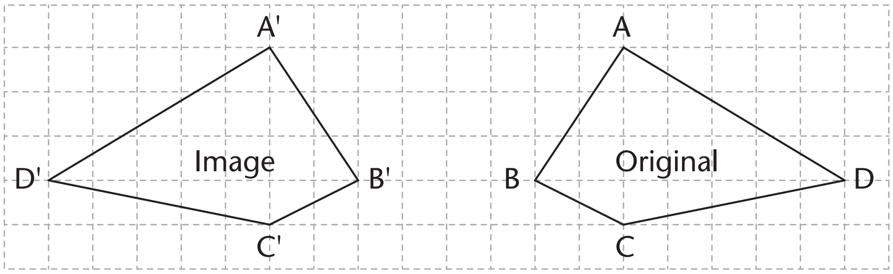
(a)



(b)

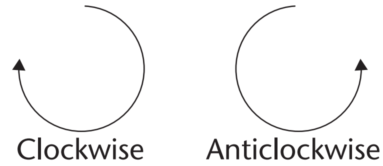


(c)



6.5 Rotating figures

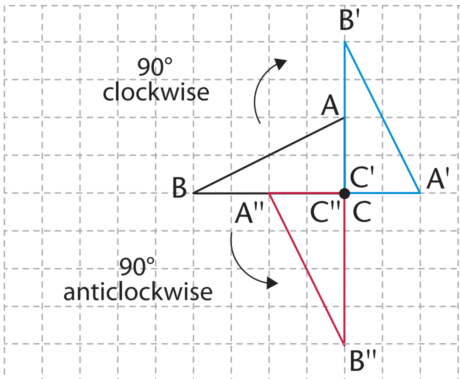
When a figure is **rotated** it is turned in a clockwise direction or in an anticlockwise direction around a particular point. This point is called the **centre of rotation** and could be inside the figure or outside of the figure.



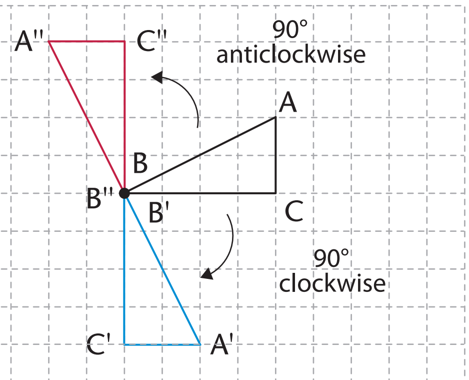
In this case, **about** means "around".

The following diagrams show ΔABC rotated 90° clockwise and 90° anticlockwise about different centres of rotation.

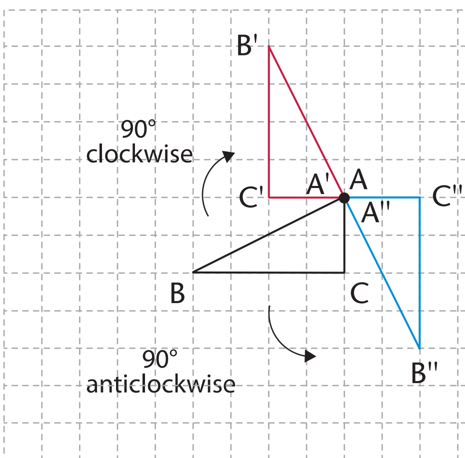
Centre of rotation is at C



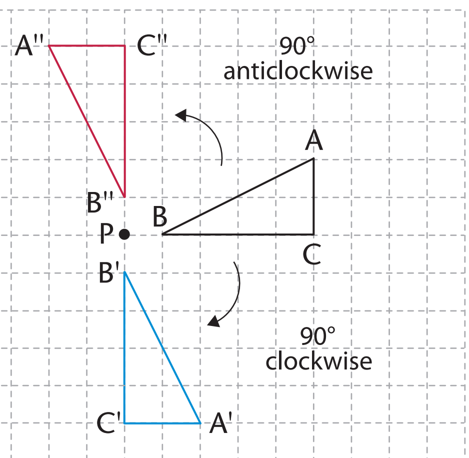
Centre of rotation is at B



Centre of rotation is at A



Centre of rotation is at P



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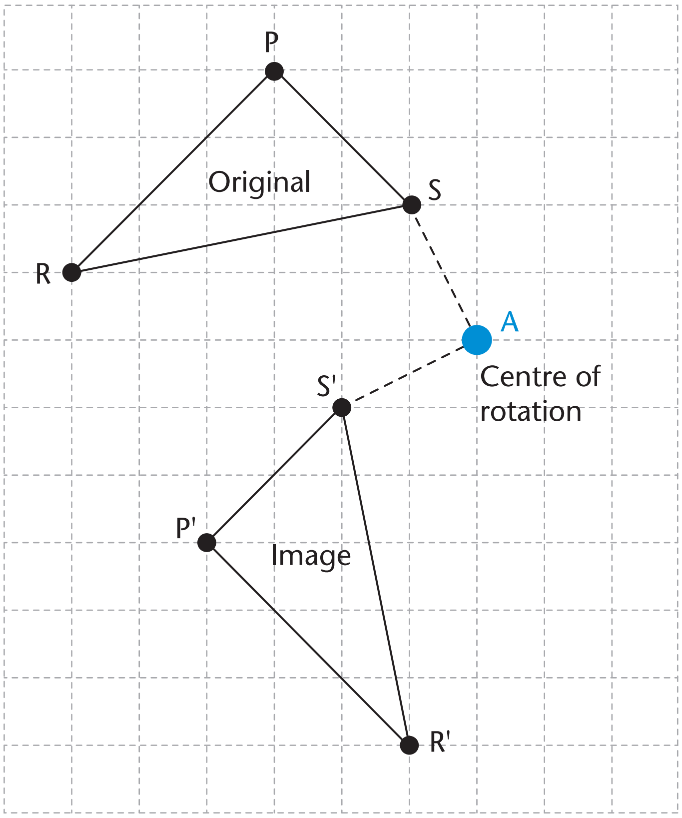
INVESTIGATING THE PROPERTIES OF ROTATION

In the following diagrams, the centre of rotation is point A. ΔPRS has been rotated anticlockwise through 90° about point A.



1. Lines have been drawn to join A to point S, and A to point S'.
   (a) Measure the distance from A to S.

   ....

   (b) Measure the distance from A to S'.

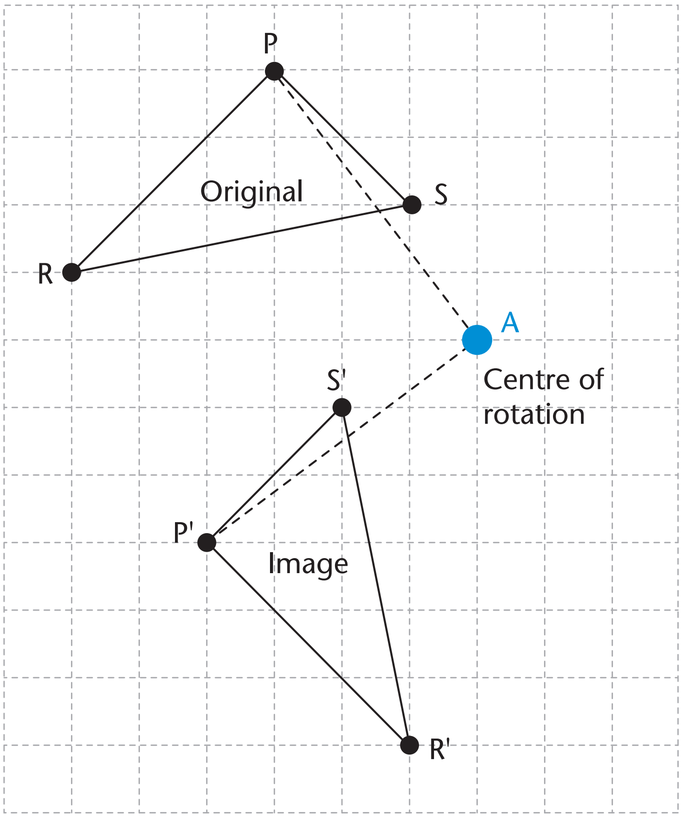
   ....

   (c) What do you notice about the distances in (a) and (b) above?

   ....

   (d) Measure the size of the angle SAS'. What do you notice?

   ....



2. Lines have been drawn to join A to P, and A to P'.
   (a) Measure the distance from A to P.

   ....

   (b) Measure the distance from A to P'.

   ....

   (c) What do you notice about the distances in (a) and (b) above?

   ....

   (d) Measure the size of the angle PAP'. What do you notice?

   ....

3. Lines have been drawn to join A to R, and A to R'.
   (a) Measure the distance from A to R.

   ....

   (b) Measure the distance from A to R'.

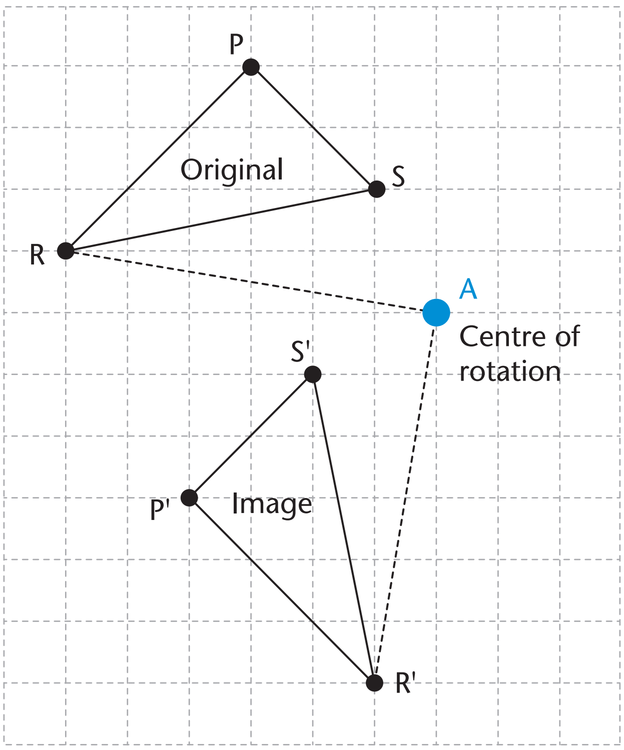
   ....

   (c) What do you notice about the distances in (a) and (b) above?

   ....

   (d) Measure the size of the angle RAR'. What do you notice?

   ....

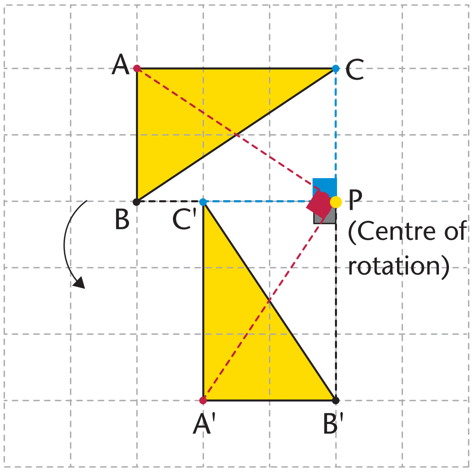


4. In any of the diagrams in questions 1 to 3 above, measure the sides of the original triangle and the corresponding sides of the image. What do you notice?

   ....

Properties of rotation

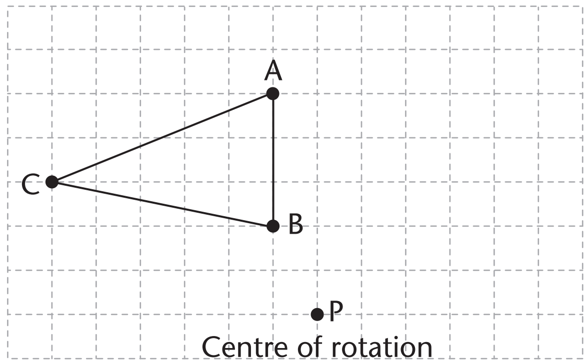
- The distance from the centre of rotation to any point on the original is equal to the distance from the centre of rotation to the corresponding point on the image. In the diagram on the right: PA = PA', PB = PB' and PC = PC'.
- The angle formed by the connecting lines between any point on the original figure, the centre of rotation and the corresponding point on the image is equal to the angle of rotation. For example, if the image is rotated through 90°, this angle will be equal to 90°. If the image is rotated through 45°, the angle will be 45°.
- When a figure is rotated, its shape and size do not change.



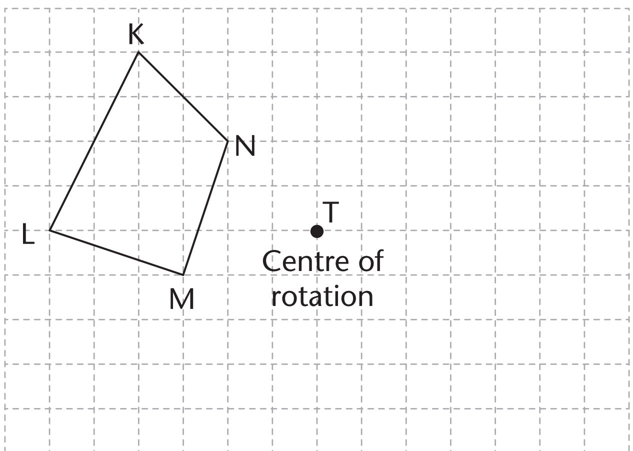
PRACTISE ROTATING FIGURES

1. Rotate triangle ΔABC 90° clockwise about point P as follows:
   (a) Plot the image of each vertex on the grid. Remember:
   - The image point must be the same distance from P as the original point.
   - The angle that is formed between the line connecting an original point to point P and the line connecting its image point to point P must be the same as the angle of rotation. In this case, it must be 90°.

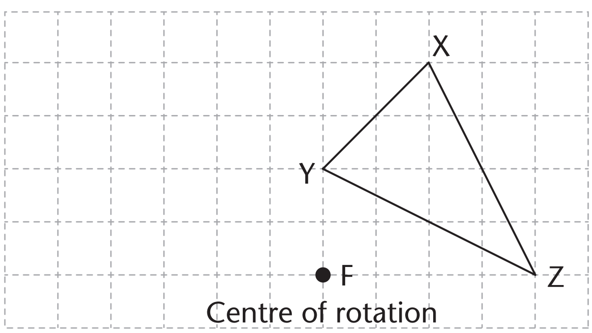
   (b) Join the image points to create ΔA'B'C'.



2. Rotate KLMN 180° about point T.



3. Rotate ΔXYZ 90° anticlockwise about point F.

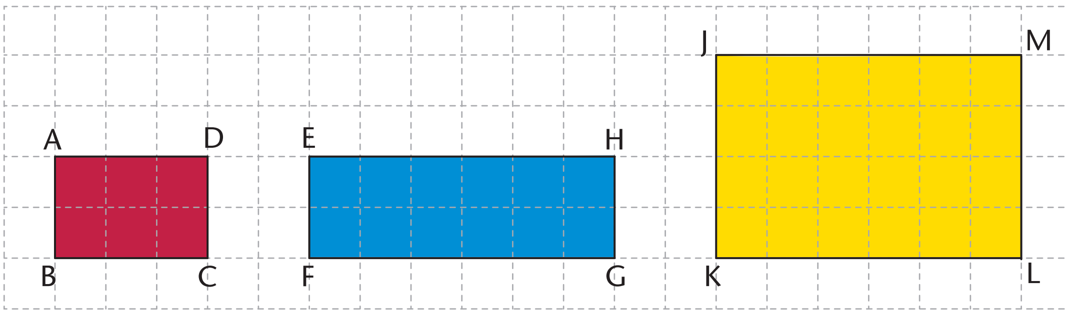


6.6 Enlarging and reducing figures

Enlarging a figure means that we make it bigger in a specific way. **Reducing** a figure means that we make it smaller in a specific way. Enlarging or reducing figures is also called **resizing**.

INVESTIGATE THE PROPERTIES OF ENLARGEMENTS AND REDUCTIONS

1. Look at the following rectangles and answer the questions below.



(a) Rectangle EFGH:

How many times is FG longer than BC? ....

How many times is EF longer than AB? ....

(b) Rectangle JKLM:

How many times is KL longer than BC? ....

How many times is JK longer than AB? ....

When the lengths of **all the sides** of a figure are **multiplied by the same number** to produce a second figure, the second figure is an **enlargement** or **reduction** of the first figure.

The number by which the sides are multiplied to produce an enlargement or reduction is called the **scale factor**. The scale factor in question 1(b) above is 2. We say that figure ABCD has been enlarged (or resized) by a scale factor of 2 to produce figure JKLM.

Figure EFGH is not an enlargement of figure ABCD because not *all* its sides have been increased by the *same* scale factor.

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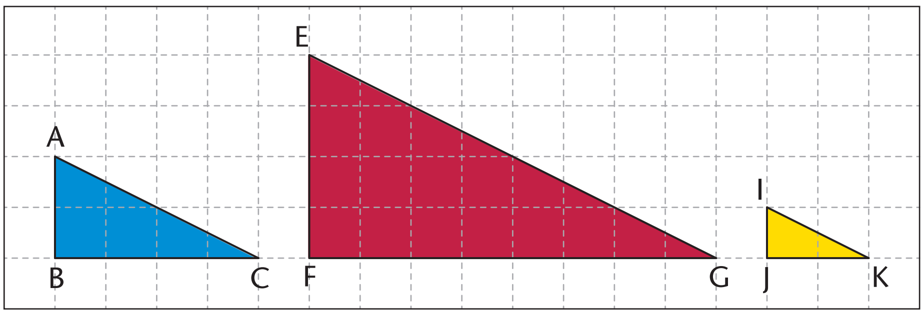
The scale factor

- When the scale factor is 1, the image is the same size as the original.
- When the scale factor is <1, the image is a reduction. For example, if the scale factor is $\frac{1}{2}$ or 0,5, each side of the image is half the length of its corresponding side in the original figure.
- When the scale factor is >1, the image is an enlargement. For example, if the scale factor is 2, each side of the image is double the length of its corresponding side in the original figure.

2. Look at the following triangles and answer the questions that follow.



(a) How many times is:

- FG longer than BC? ......
- EF longer than AB? ......
- EG longer than AC? ......
- JK shorter than BC? ......
- IJ shorter than AB? ......
- IK shorter than AC? ......

(b) Is ΔEFG an enlargement of ΔABC? Explain your answer.

....

(c) Is ΔIJK a reduction of ΔABC? Explain your answer.

....

Similar figures

When figures are enlarged or reduced, the enlarged or reduced image is **similar** to the original figure. ΔABC, ΔEFG and ΔIJK above are all similar. We also say that the lengths of their corresponding sides are **in proportion**.

If two or more figures are **similar**:

- their corresponding angles are equal, and
- their corresponding sides are longer or shorter by the same scale factor.

PRACTISE RESIZING FIGURES

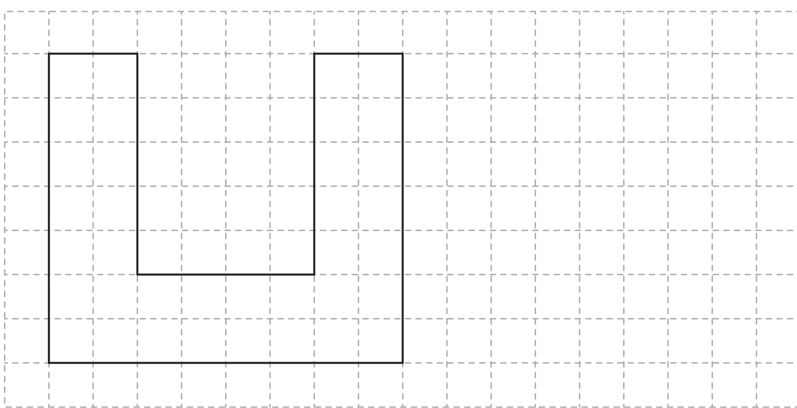
1. State whether the following scale factors will produce a larger or smaller image:

   (a) 5 .... (b) 0,25 ....

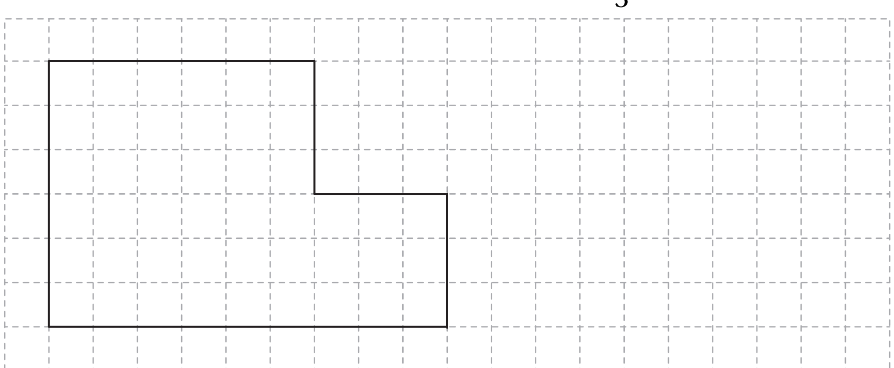
   (c) 1,2 .... (d) $\frac{3}{8}$ ....

2. Enlarge the triangle below with a scale factor of 2.

3. Resize the following figure. Use a scale factor of 0,5.

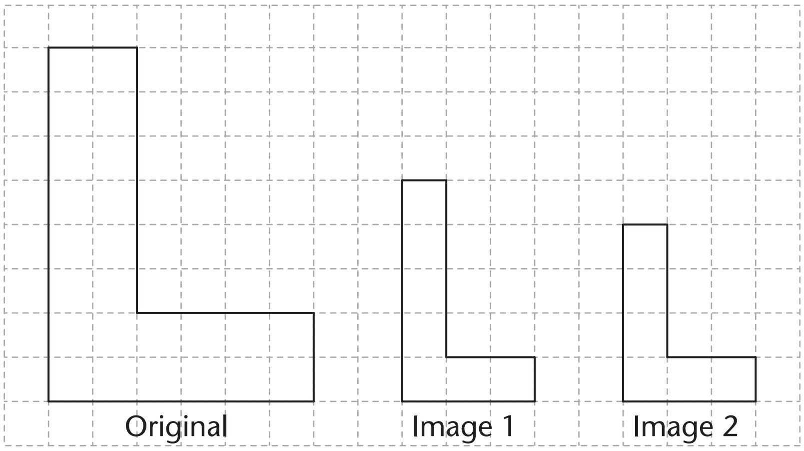


4. Resize the figure below. Use a scale factor of $\frac{1}{3}$.



5. (a) Which image below is similar to the original? ....

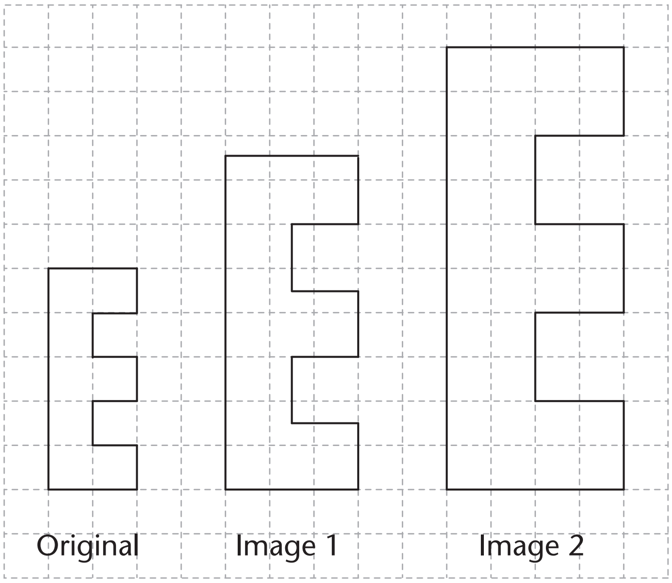
(b) State the scale factor by which it has been resized. ....



6. What scale factors were used to produce image 1 and image 2 from the original?

Image 1: ....

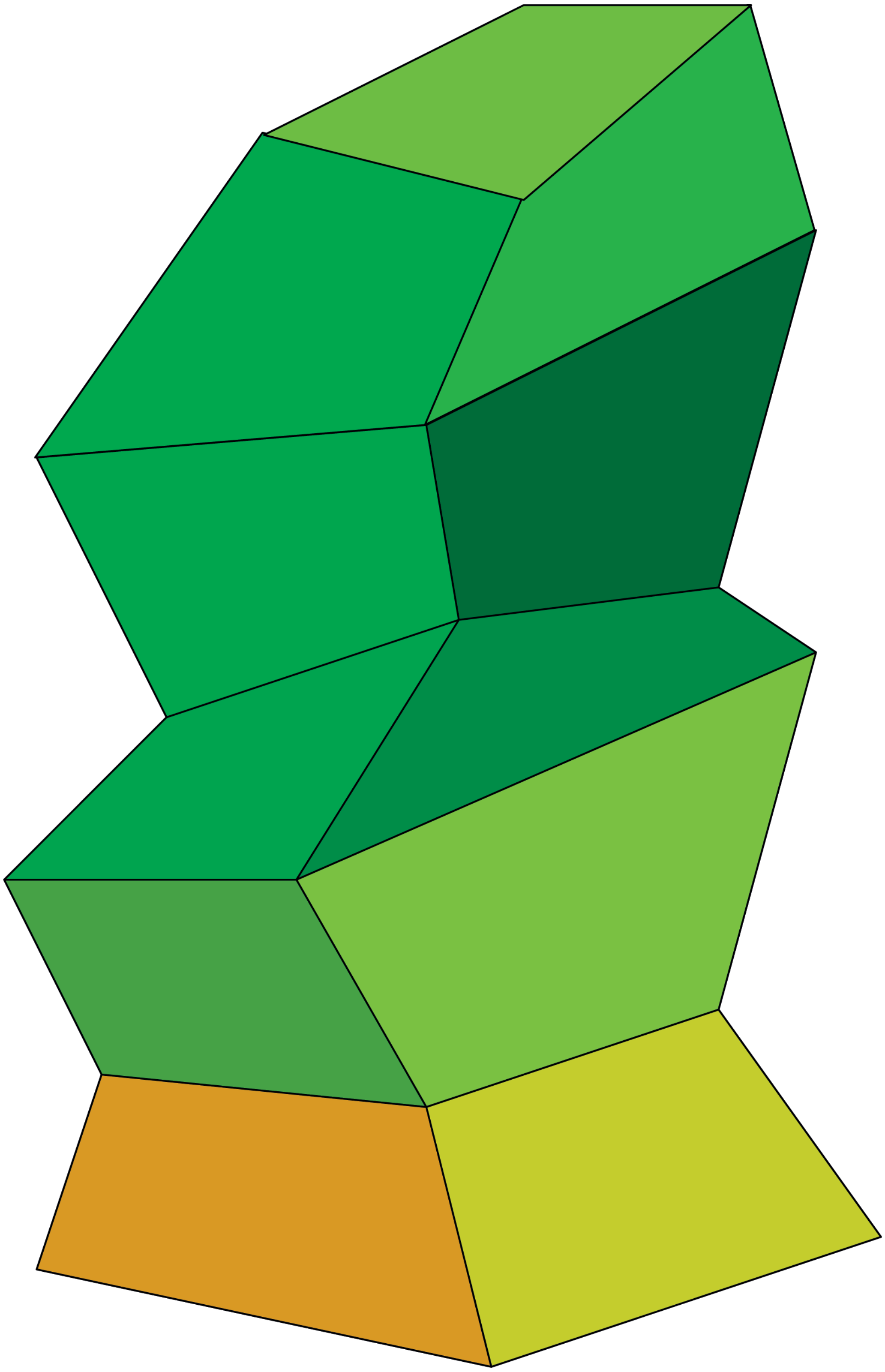
Image 2: ....



Chapter 7
Geometry of 3D objects

In this chapter, you will revise the work you have done on 3D objects in Grade 6. This includes describing, sorting and comparing 3D objects by focusing on the number and shapes of their faces, their number of vertices and their number of edges. The 3D objects you will work with in this chapter are cubes, rectangular prisms, triangular prisms, pyramids and cylinders.

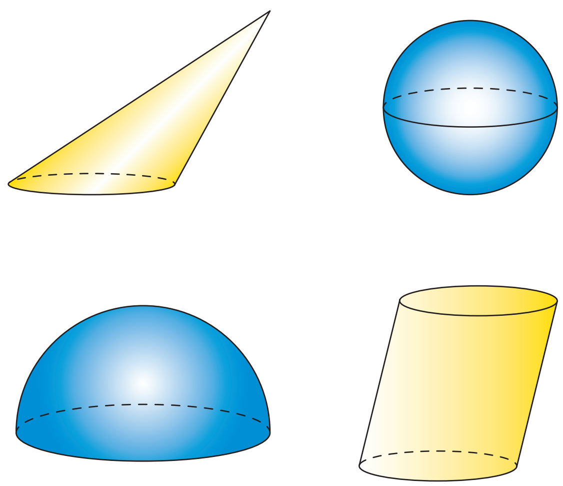
After classifying various 3D objects, you will build cardboard or paper models of different cubes and prisms. In order to do this, you will need to know how to draw nets (or flat patterns) for these 3D objects. As you do the drawings of the nets of the 3D objects and as you construct the objects, you will find that you have to think carefully about which sides of the shapes in the net have to match up. This will give you clues about how long each of these sides has to be.



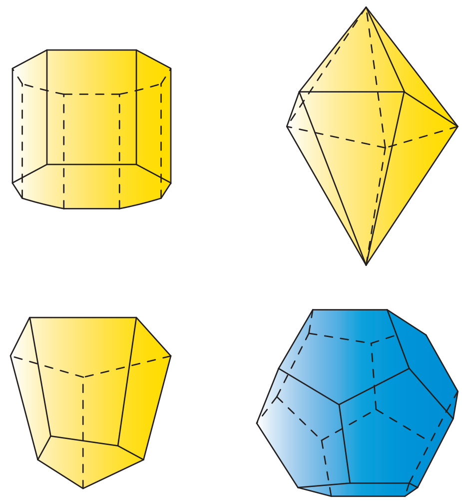
7 Geometry of 3D objects

7.1 Classifying 3D objects

There are two main groups of objects with **three dimensions** (length, width and height), namely those with **curved surfaces** and those with **flat surfaces**. **Spheres** (balls), **cylinders** and **cones** are examples of objects with curved surfaces. Objects with only flat surfaces are called **polyhedra**.



Examples of objects with curved surfaces



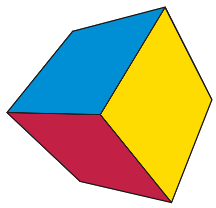
Examples of objects with flat surfaces only

WHAT IS A POLYHEDRON?

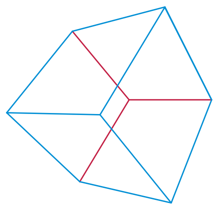
A **polyhedron** is a three-dimensional object (or 3D object) made of flat surfaces only. It has no curved surfaces. It consists of faces, edges and vertices.

A **face** is the flat surface of a 3D object.
An **edge** is the segment where two faces of a polyhedron intersect.
A **vertex** is the point where the edges meet.

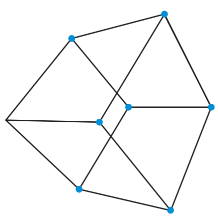
We say: one **polyhedron**; two or more **polyhedra**.
We say: one **vertex**; two or more **vertices**.



Faces



Edges



Vertices

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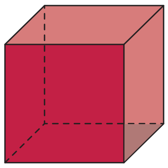
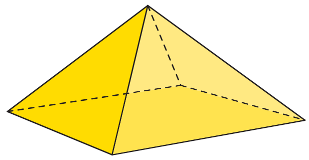
A **cube** has 6 faces, 12 edges and 8 vertices.

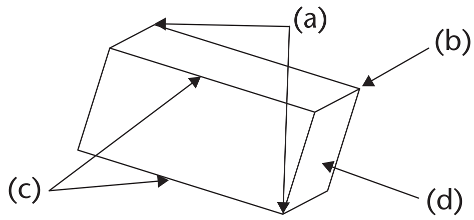


This **pyramid** has 5 faces, 8 edges and 5 vertices.



IDENTIFYING AND DESCRIBING 3D OBJECTS

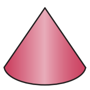
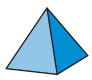
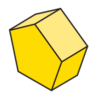
1. Identify parts (a) to (d) on the figure correctly.



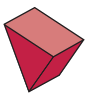
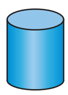
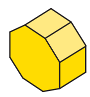
(a) .... (b) ....

(c) .... (d) ....

2. Which of the following objects are polyhedra?

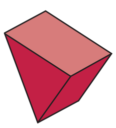
(a)  (b)  (c) 

....

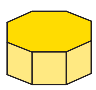
(d)  (e)  (f) 

....

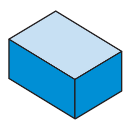
3. How many faces, edges and vertices does each of the following polyhedra have?

(a) 

Faces: ....
Edges: ....
Vertices: ....

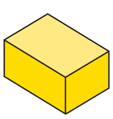
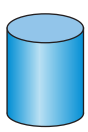
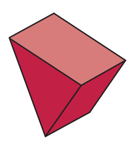
(b) 

Faces: ....
Edges: ....
Vertices: ....

(c) 

Faces: ....
Edges: ....
Vertices: ....

4. Six learners each used play dough to make a 3D object. Use the descriptions on the next page to match each 3D object to the learner who made it.

(a)  (b)  (c) 

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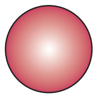
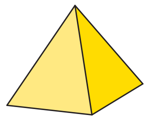
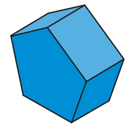
(d) 

(e) 

(f) 

....

- Tumi's object has 6 vertices and 5 faces.
- Debbie's object has 8 vertices and 12 edges.
- Brad made an object that has 7 faces and 10 vertices.
- Xola made an object with no vertices.
- Mpuka's object has 8 edges and 5 faces.
- Maggie made an object with 2 circles and no vertices.

7.2 Prisms and pyramids

DIFFERENCE BETWEEN PRISMS AND PYRAMIDS

Prisms and **pyramids** are two special groups of polyhedra.

Prisms

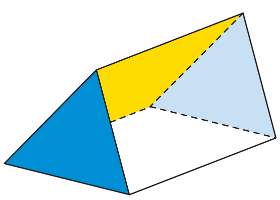
A **prism** is a polyhedron with two faces that are congruent and parallel polygons. These faces are called **bases** and they are connected by **lateral faces** that are parallelograms.

Congruent means exactly the same shape and size. **Lateral faces** are faces that aren't bases.

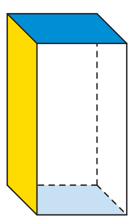
In the case of **right prisms** the bases are connected by rectangles which are perpendicular to the base and the top. This means the lateral faces of a right prism make a 90° angle with the bases.

A prism is named according to the shape of its base. So a prism whose base is a triangle is called a triangular prism; a prism whose base is a rectangle or square is called a rectangular prism; and a prism with a pentagonal base is called a pentagonal prism.

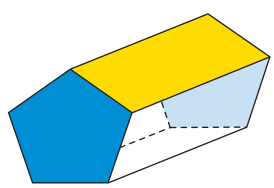
Any pair of faces in a prism that are congruent and parallel can be the bases of that prism. A cube is a special type of prism. It has 6 congruent faces; therefore any of its faces can be a base.



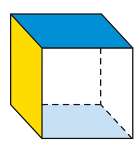
Triangular prism



Rectangular prism

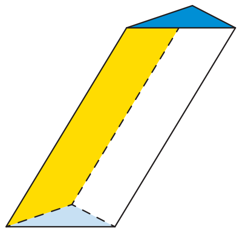


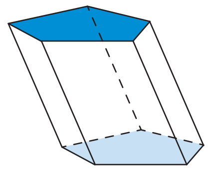
Pentagonal prism



Cube

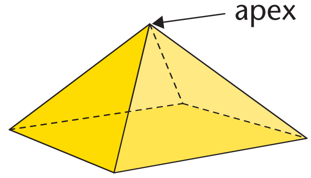
These are two **oblique prisms** – the one on the left is a triangular prism and the one on the right is an oblique pentagonal prism.





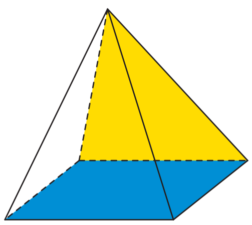
Pyramids

A **pyramid** has only one base. The lateral faces of a pyramid are always triangles. These triangles meet at the same vertex at the top. This vertex is called the **apex** of the pyramid. In a **right pyramid**, the lateral faces are isosceles triangles.

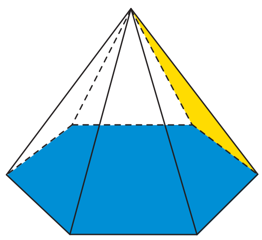


There are many types of pyramids. The type of pyramid is determined by the shape of its base. For example, a triangular-based pyramid has a triangle as its base, a square-based pyramid has a square as its base and a hexagonal-based pyramid has a hexagon as its base.

A triangular-based pyramid is also called a **triangular pyramid**; a square-based pyramid is also called a **square pyramid**; a hexagonal-based pyramid is also called a **hexagonal pyramid**, etc.

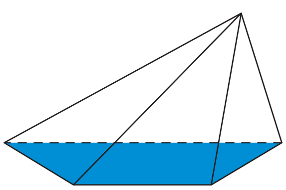


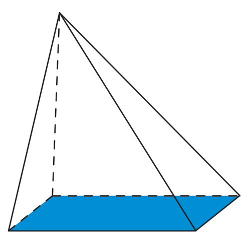
Square-based pyramid
It has 5 faces:
1 square,
4 triangles



Hexagonal-based pyramid
It has 7 faces:
1 hexagon,
6 triangles

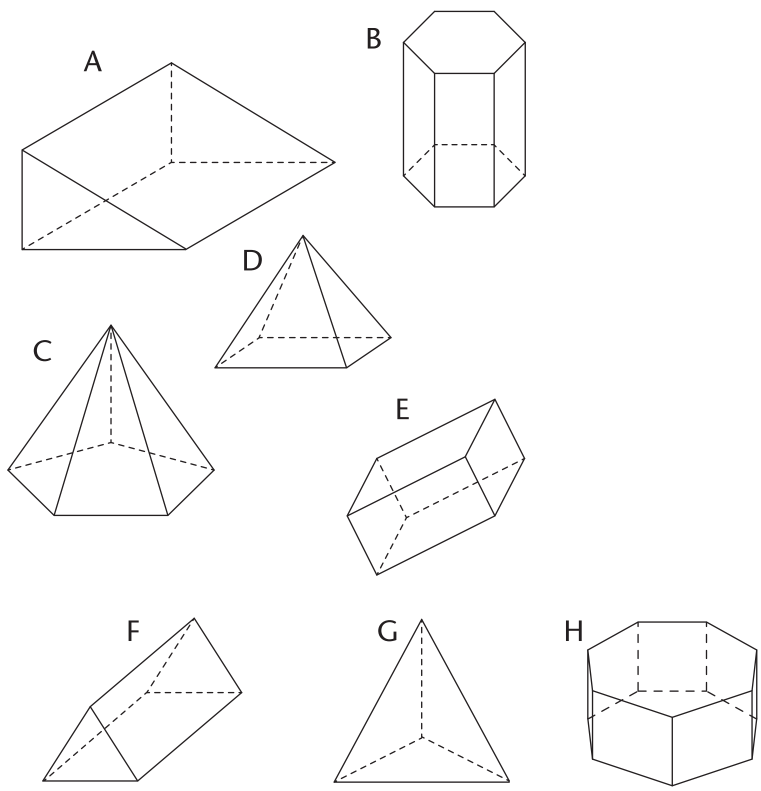
If a pyramid is not a right pyramid, it is called an **oblique pyramid**, like the two shown below. The lateral faces of an oblique pyramid are not necessarily isosceles triangles.





IDENTIFYING PRISMS AND PYRAMIDS

1. Shade the base of each of the following figures and write down whether it is a prism or a pyramid. In some cases there is more than one possibility for the base.



A: ....

B: ....

C: ....

D: ....

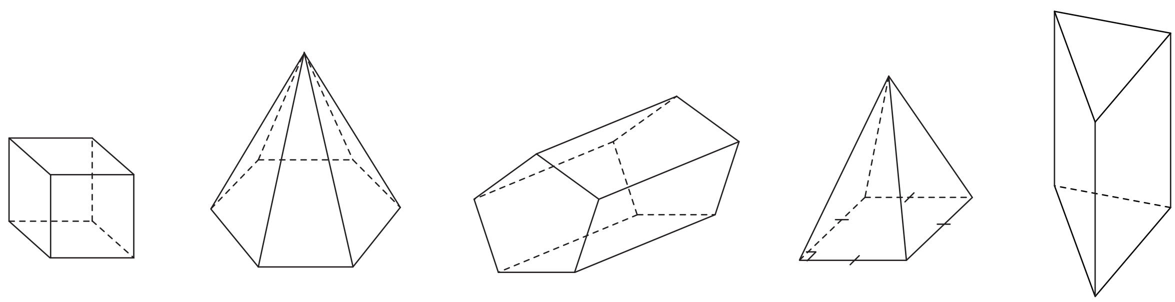
E: ....

F: ....

G: ....

H: ....

2. Join each 3D object with its correct name.

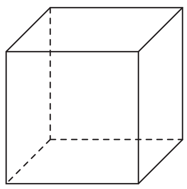


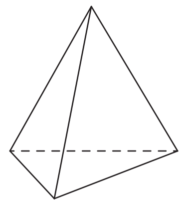
Hexagonal-based pyramid	*Triangular prism*	*Square-based pyramid*	*Cube*	*Pentagonal prism*

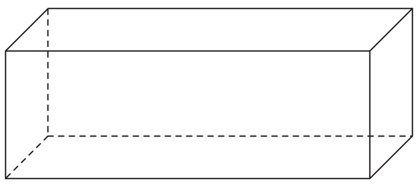
7.3 Describing, sorting and comparing 3D objects

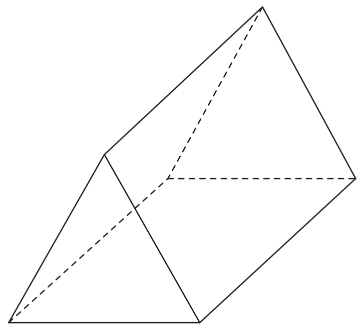
PRACTISE DESCRIBING AND CLASSIFYING 3D OBJECTS

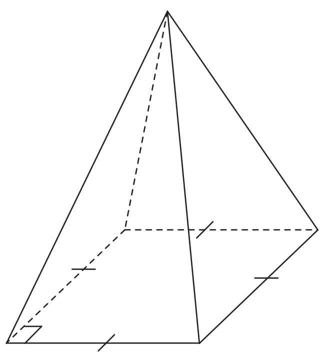
1. For each of the following 3D objects:
   (a) name the object, and
   (b) describe the number of faces it has and the shapes of these faces.

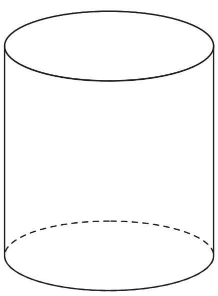












2. (a) Sort the 3D objects below into prisms, pyramids and cylinders by writing down the correct letters.

Prisms: ....

Pyramids: ....

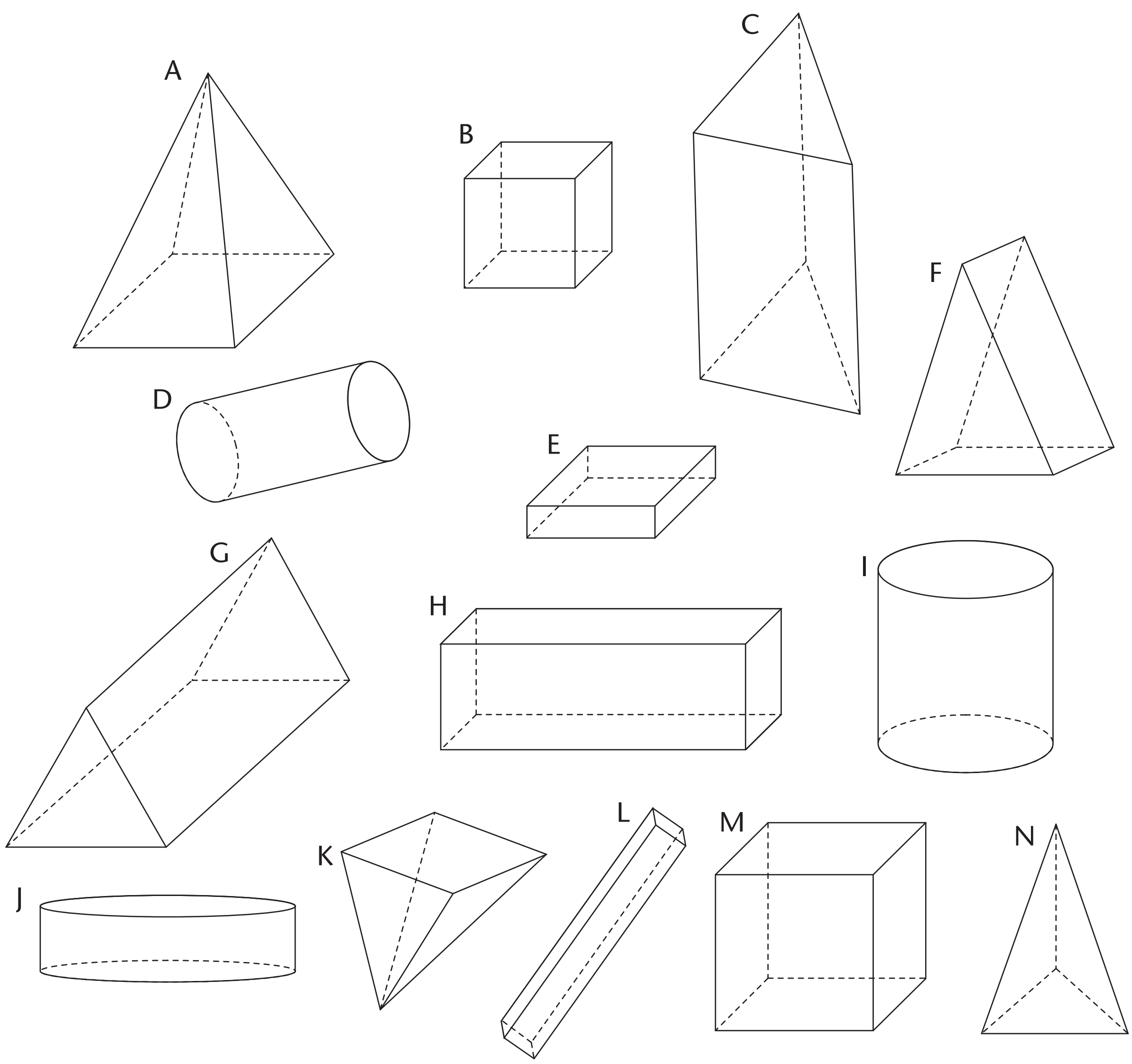
Cylinders: ....

(b) Further divide the prisms into three groups and name each group by writing down the letters.

Cubes: ....

Rectangular prisms: ....

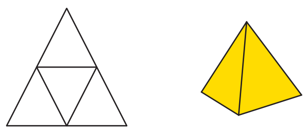
Triangular prisms: ....



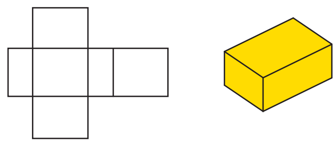
7.4 Nets of 3D objects

WHAT IS A NET?

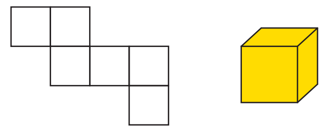
In mathematics, a **net** is a flat pattern that can be folded to form a 3D object. Different 3D objects have different nets. Sometimes the same 3D object can have different nets. Here are examples of different 3D objects and their nets.



Triangular pyramid



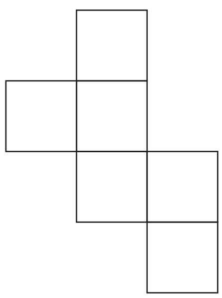
Rectangular prism



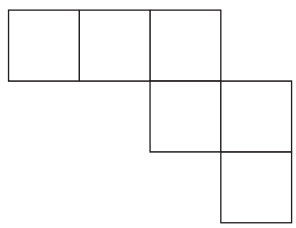
Cube

In this section, you are going to focus on the net of a cube. In order for a net to form a cube, it must consist of 6 equal squares. But not all net patterns that consist of 6 squares will fold into a cube.

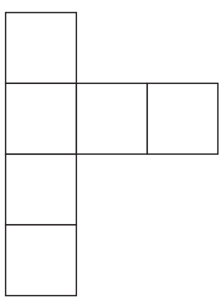
Only one of the nets below will fold into a cube. Write down which one it is. ....



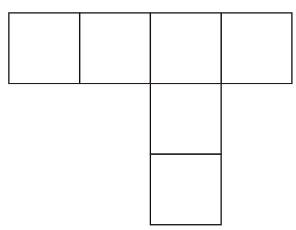
A



B



C

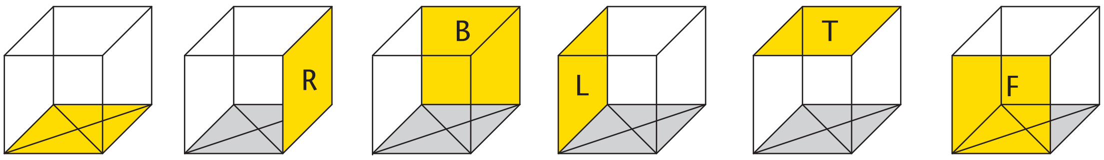


D

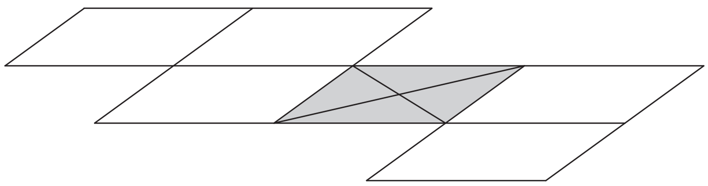
A CUBE OR NOT A CUBE

In order to decide whether or not a net will fold into a cube, you have to imagine what will happen when you fold the net. Read through the following steps.

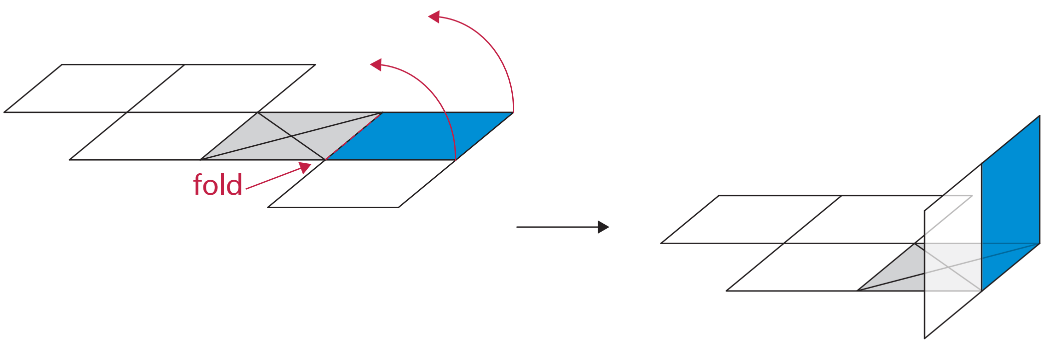
1. We can label the faces of a cube: bottom face (X), right face (R), back face (B), left face (L), top face (T) and front face (F). We will use these terms in the rest of the steps.



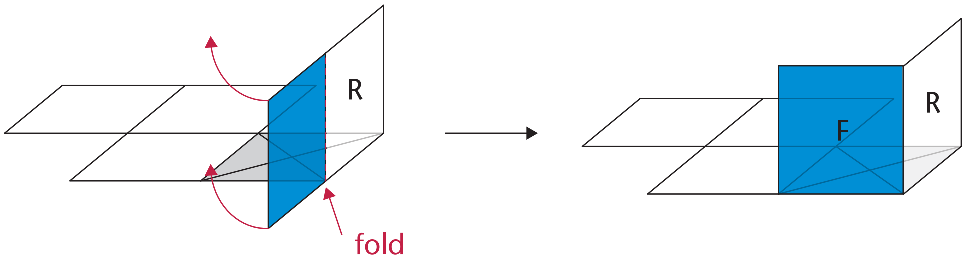
2. Start by choosing one square of the net as the bottom face (marked with an X).



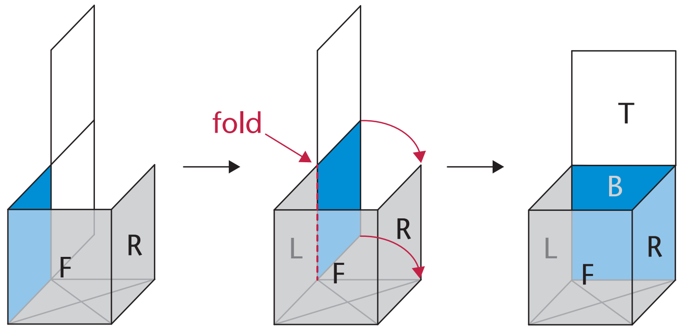
3. Look at the square to the right of the X. (It is coloured blue.) If you fold the net on the red line, the blue square will be the right face of the cube.



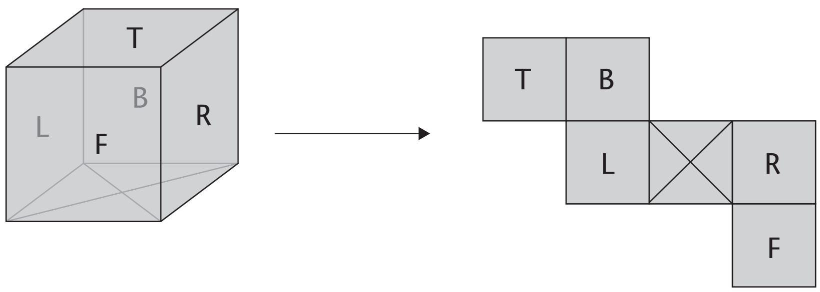
4. Look at the next square to the left of the right face. If you fold this square on the red line, it will become the front face of the cube.



5. Look at the square to the left of the front face. It will fold to become the left face of the cube.
6. The square to the left of the left face of the cube will become the back face of the cube.
7. The last square will form the top.



8. Therefore you can label the squares on the net as follows:

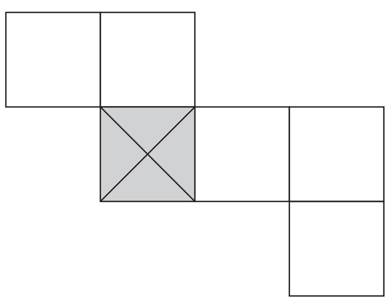


9. Since each square on the net corresponds with a face of the cube, this net can be folded into a cube.

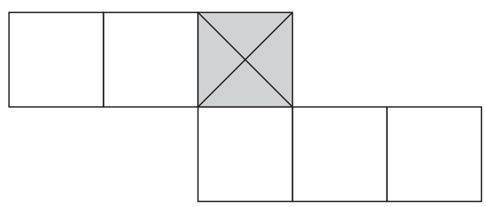
IDENTIFYING THE NETS OF A CUBE

For each of the following nets, determine whether it will fold into a cube or not by labelling the squares to match the faces of a cube.

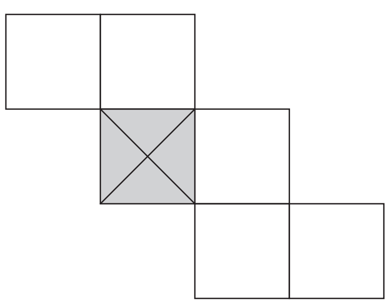
1. ....



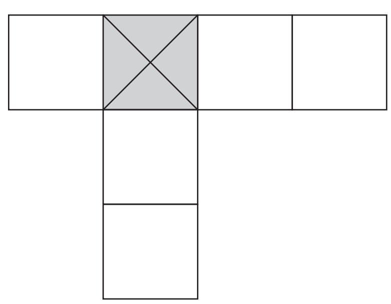
2. ....



3. ....



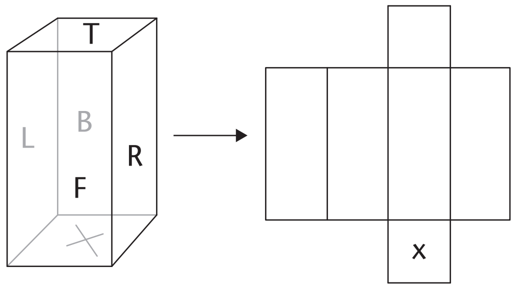
4. ....



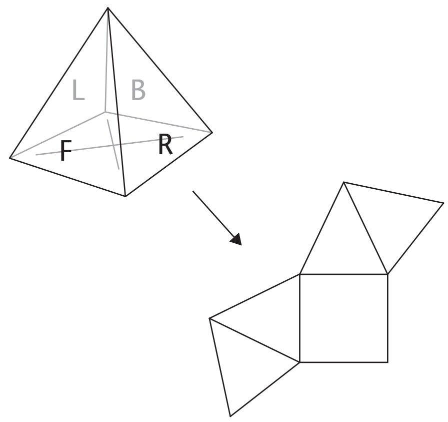
NETS OF OTHER 3D OBJECTS

1. In each of the following cases, label the faces on the net according to the labels on the 3D object.

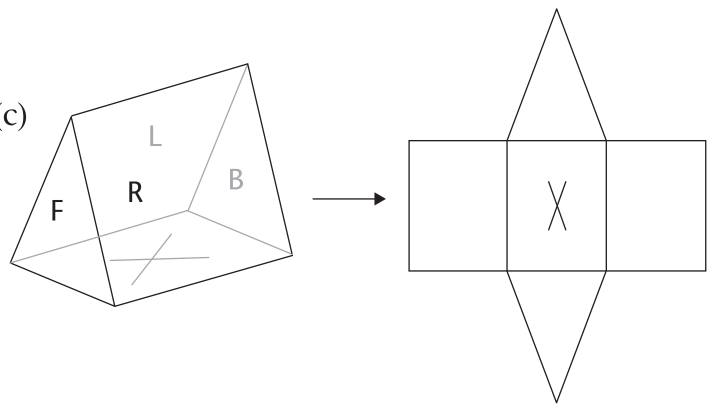
(a)



(b)

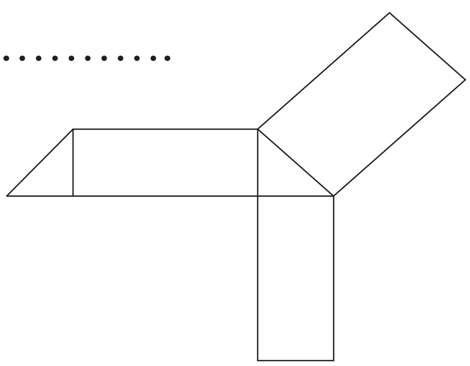


(c)

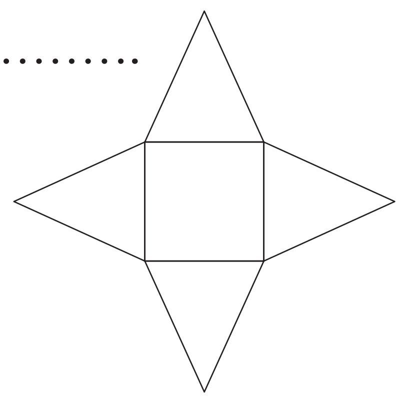


2. Decide whether the following nets will form 3D objects.

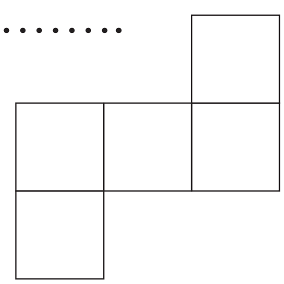
(a) ....



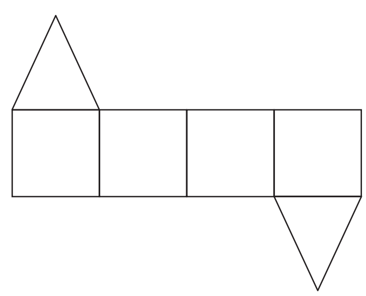
(b) ....



(c) ....

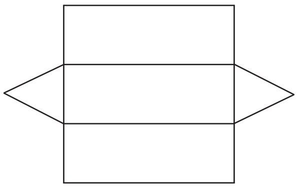


(d) ....

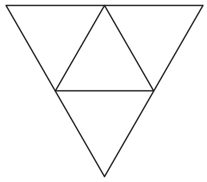


3. Identify the following nets:

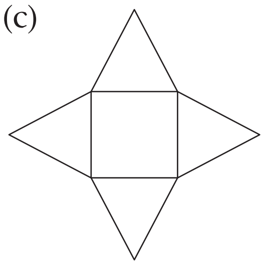
(a)



(b)

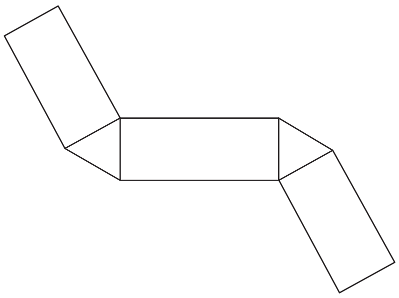


(c)

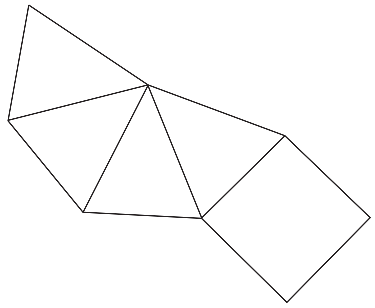


....

(d)

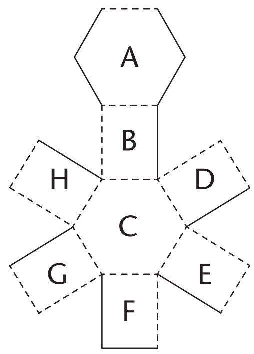


(e)



....

4. (a) Identify the shapes that the net on the right consists of.



A: ....

B: ....

C: ....

D: ....

E: ....

F: ....

G: ....

H: ....

(b) How many rectangular faces does the net have? ....

(c) How many other shapes does the net have? What are they? ....

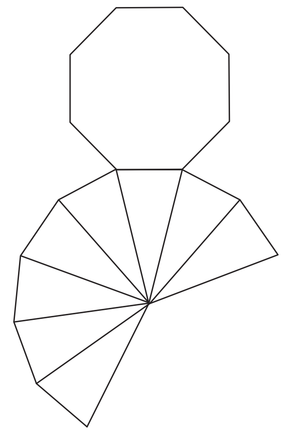
(d) Will this form a pyramid or a prism? ....

(e) How do you know? ....

....

(f) Name the 3D object that the net will form. ....

5. Answer the following questions about this net.



(a) Which shapes does this net consist of?

....

(b) How many triangular faces does the net have?

....

(c) How many other shapes does the net have?

....

(d) Will this form a pyramid or a prism?

....

(e) How do you know?

....

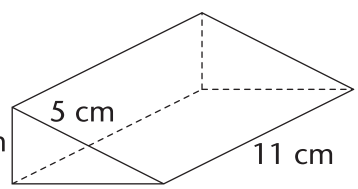
(f) Name the 3D object that the net will form.

....

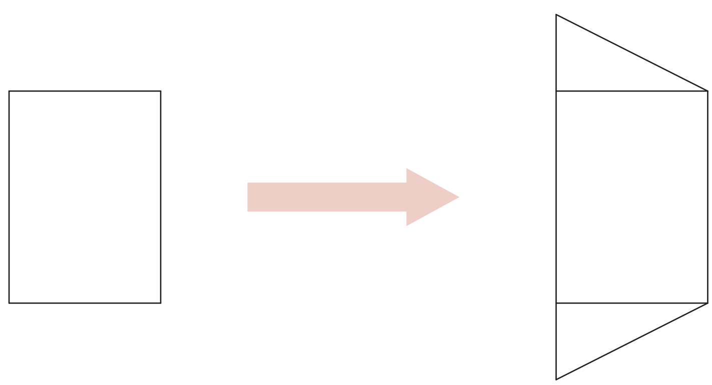
7.5 Using nets to construct cubes and prisms

HOW TO DRAW A NET OF A PRISM

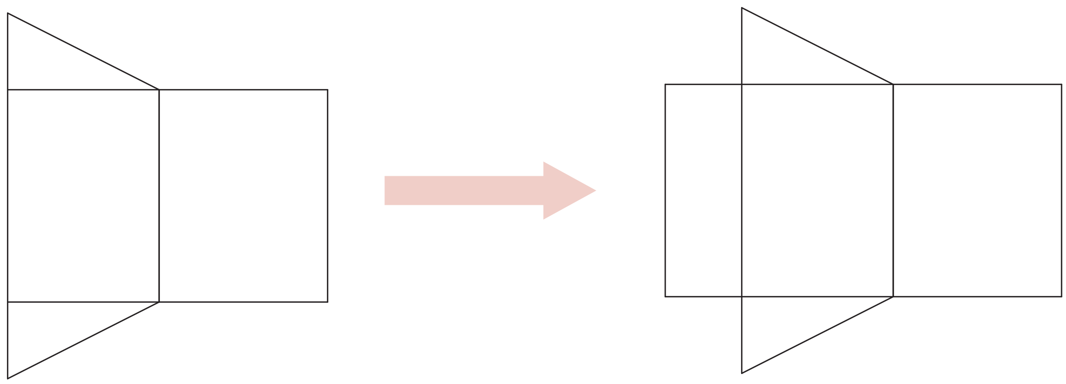
Look at the triangular prism alongside. Its faces are 2 right-angled triangles and 3 rectangles. (Since the base has 3 edges, there will be 3 rectangles.)



1. Draw a rectangle, which will be the bottom of the prism. Then add a right-angled triangle at the top and a mirror image of the triangle at the bottom of the rectangle.



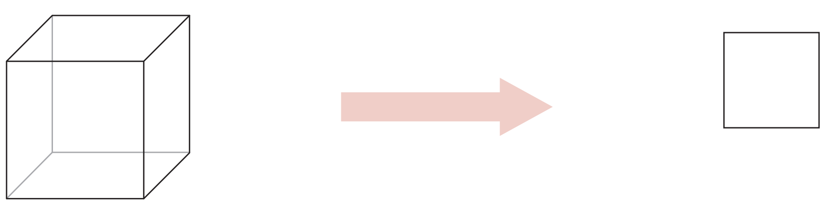
2. Draw the other two rectangles next to the centre one. The rectangle on the right must fit onto the side opposite the right angle in the triangle when it is folded, so that rectangle will be the biggest. The rectangle on the left is the smallest of the three rectangles.



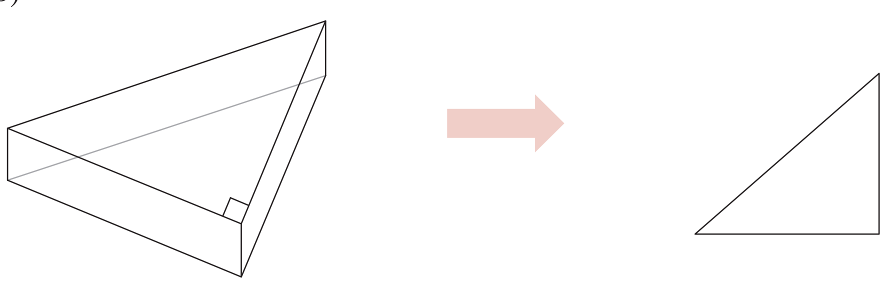
PRACTISE DRAWING NETS AND CONSTRUCTING 3D MODELS

1. Complete the following nets:

(a)



(b)



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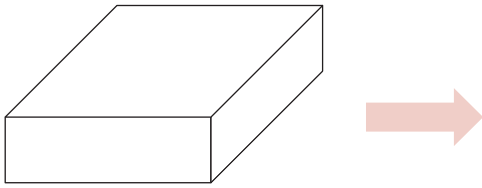
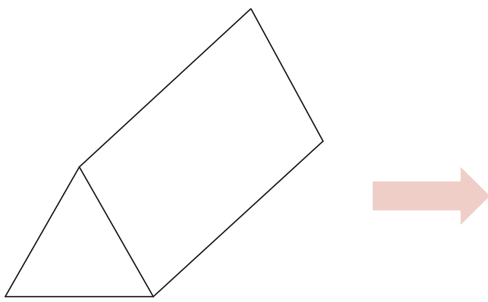
2. Draw nets for the following objects:

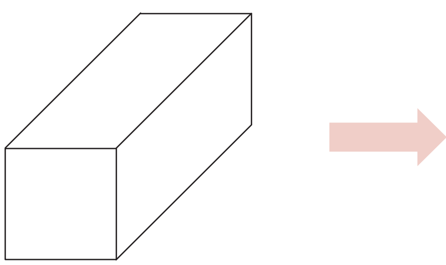
(a)



(b)



(c)



3. (a) Copy each of the nets that you drew in questions 1 and 2 above onto cardboard or paper.
   (b) Cut out the nets, and fold and paste them to make each 3D object.
   (c) Write down what you found difficult in making your 3D models and how you overcame this difficulty.

....

....

....

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Term 3
Revision and assessment

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Revision

Show all your steps of working.

NUMERIC AND GEOMETRIC PATTERNS

1. For each of the following sequences, (i) describe in words the relationship between the terms in the sequence, and (ii) use the relationship to find the next three terms in the sequence.

   (a) 23; 19; 15; ...

   ....

   ....

   (b) 0,1; 0,2; 0,4; 0,8; ...

   ....

   ....

   (c) $\frac{1}{2}$; $\frac{3}{2}$; $2\frac{1}{2}$; ...

   ....

   ....

   (d) 1; 4; 9; 16; ...

   ....

   ....

   (e) 2; 4; 7; 11; ...

   ....

   ....

   (f) 2; 9; 28; 65; ...

   ....

   ....

   (g) 21 200; 2 120; 212; ...

   ....

   ....

2. Write down the first three terms of a sequence that fits the description given:
   (a) Each term is 2,3 bigger than the previous term.

   ....

   (b) Each term is $\frac{1}{3}$ smaller than the next term.

   ....

   (c) Each term is half the previous term.

   ....

3. (a) Write down the values of *a* to *d*:

Term number	1	2	3	4	5	10
Value of the term	7,2	7,7	8,2	*a*	9,2	*b*

....

....

Term number	1	2	3	*c*	*d*	7
Value of the term	1	3	9	81	243	729

....

....

   (b) Explain how you obtained the values of *b* and *d*.

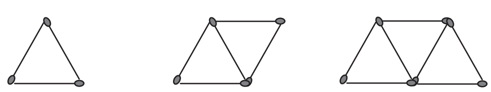
....

....

....

....

4. Below are the first three arrangements in a pattern created with matches.



1 2 3

   (a) Complete the table below:

Number of the arrangement	1	2	3	4	15	
Number of matches needed	3	5				121

   (b) Write in words the rule that describes the number of matches needed for each new arrangement.

....

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FUNCTIONS AND RELATIONSHIPS 1

1. Use the formula for the area of a rectangle ($A = l \times b$) to calculate the following:
   (a) The area, if the length is 0,4 m and the breadth is 0,3 m

   ....

   ....

   (b) The length, if the area is 12,4 cm^2 and the breadth is 4 cm

   ....

   ....

   (c) The breadth, if the area is 14,4 m^2 and the length is 12 m

   ....

   ....

2. The formula for the total surface area of a cylinder of base radius r and height h is:
   $A = 2 \times \pi \times r^2 + 2 \times \pi \times r \times h$

   π or pi (pronounced "pie") is a number approximately equal to $\frac{22}{7}$. You will learn about pi in Grade 8.

   Use this formula to calculate the surface area if $\pi = \frac{22}{7}$, the base radius is 7 cm and the height is 3 cm.

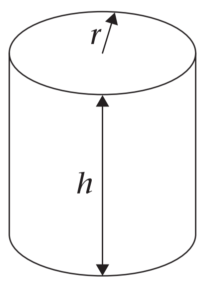
   ....

   ....

   ....

   ....

   ....



3. The formula for finding the temperature in degrees Celsius (°C) is $C = \frac{5}{9} \times (F - 32)$, where F is the temperature in degrees Fahrenheit (°F). What is the temperature in °C if it is 59 °F?

   ....

   ....

   ....

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ALGEBRAIC EXPRESSIONS 1

1. If Skumbuzo is x years old, write down in terms of x the ages of the following people:
   (a) Suzie, who is 2 years older than Skumbuzo ....
   (b) Mohau, who is two years older than Suzie ....
   (c) Lintle, who is twice as old as Skumbuzo ....

2. The relationship between a girl's age (y years old) and that of her mother is $y + 27$. How old is the girl's mother when the girl is 13 years old?

   ....

3. Write numbers in the boxes that make the statements true. All the numbers that you fill in should be decimal numbers.
   (a) If $x = 4{,}5$, then $x + 7{,}7 = \square$
   (b) If $x = 2{,}6$, then $2 \times x = \square$
   (c) If $x = \square$, then $\frac{x}{3} = \square$

4. Match each computational instruction to the correct expression. The first one has been done for you.

	$5 - y$
Add y to 5 → $5 + y$	$\frac{5}{y}$
Subtract 5 from y	$5 + y$
Multiply y by 5	$y - 5$
Divide 5 by y	$\frac{y}{5}$
Subtract 5 from y and multiply the answer by 5	y^3
Cube y	$5 \times y$
	y^2
	$5 \times (y - 5)$

ALGEBRAIC EQUATIONS 1

1. Write an equation (an open number sentence) that fits the given description:
   (a) Christian is x years old, his best friend Refilwe is y years old, and the sum of their ages is 27.

   ....

   (b) Car a is R5 000 cheaper than car b.

   ....

2. Here is an equation: $3 + c = d$
   (a) Write down a pair of numbers that makes the equation true.

   ....

   (b) Write down a different pair of numbers that makes the equation true. The numbers should be common fractions.

   ....

3. Solve for x:
   (a) $x - 6 = 15$ (b) $3 \times x = 45$ (c) $\frac{80}{x} = 4$

   ....

   ....

4. You are given that $3 \times x + 5 = 11$. Write down the value of:
   (a) $3 \times x + 4$ (b) $(3 \times x + 5)^2$

   ....

   ....

   ....

5. If $d = c^3 + 12$, calculate the value of d when c has a value of:
   (a) 3 (b) 6

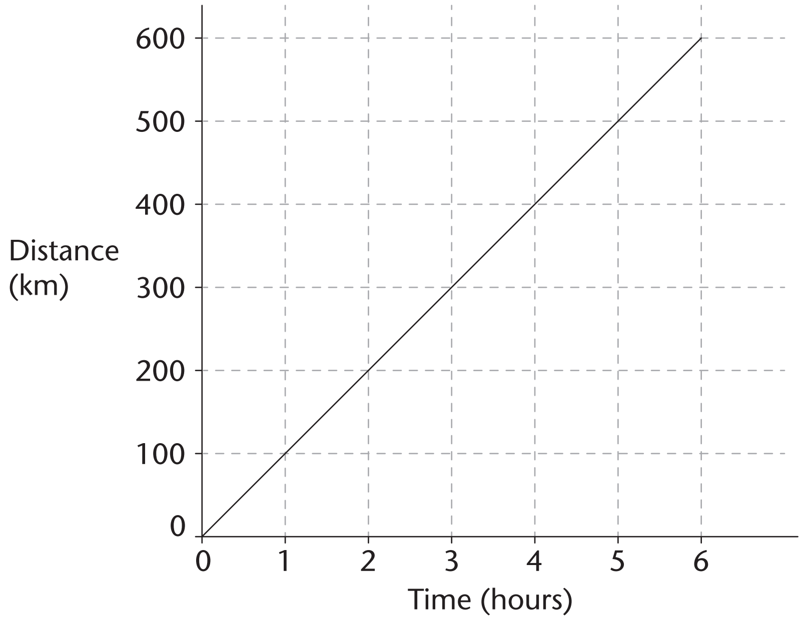
   ....

   ....

   ....

GRAPHS

1. Study the following graph, showing the distance travelled by a car on the N1, and then answer the questions that follow:



(a) Ahmed says, "This is a linear graph." Is he correct? Explain your answer.

....

(b) Sindi says the graph is increasing. Is she correct? Explain your answer.

....

....

....

(c) How far has the car travelled after 1,5 hours?

....

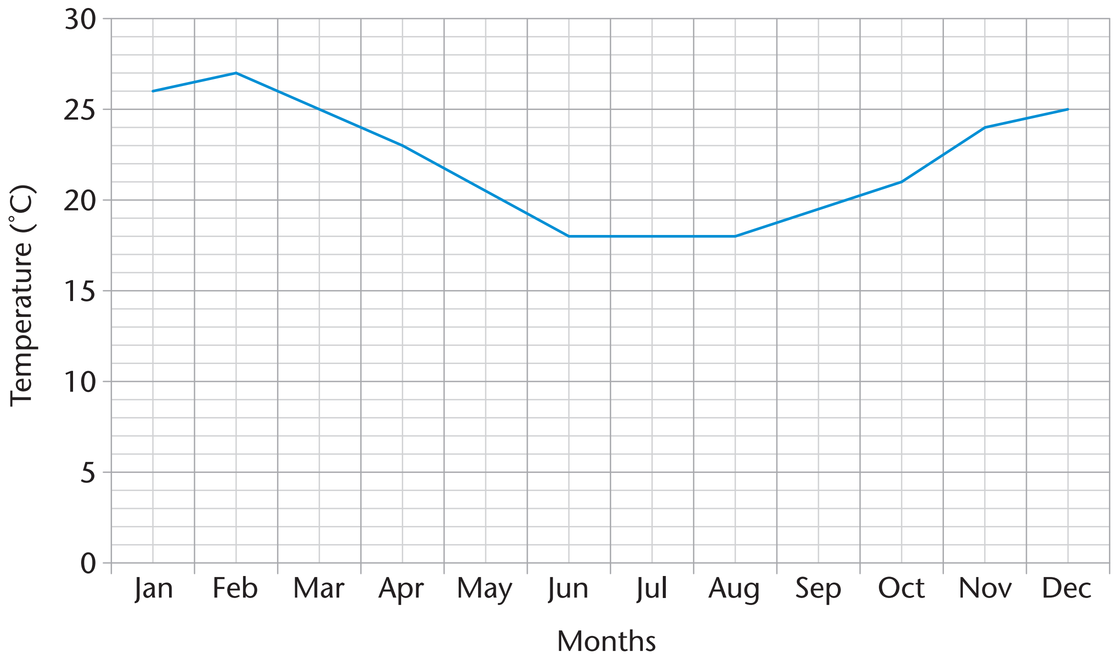
(d) Complete the following table of values by reading off from the graph:

Time (hours)	1	2	3	4	5	6
Distance (km)						

(e) If the car continued in the same way as shown on the graph, how far will it have travelled after 10 hours?

....

2. Study the following graph, showing the average maximum temperatures in Cape Town, and then answer the questions that follow:



(a) Describe the trend in the maximum temperatures from February to June.

....

(b) Describe the trend in the maximum temperatures from June to August.

....

(c) Describe the trend in the maximum temperatures from August to October.

....

(d) Which is the hottest month in Cape Town?

....

What is the average maximum temperature in this month?

....

(e) Write down the names of the coldest months in Cape Town.

....

What is the average maximum temperature during these months?

....

3. Draw graphs that fit each of the following descriptions. Label the axes. You don't have to show any values on the axes.
   (a) A speed-time graph for a vehicle travelling at a constant speed
   (b) A temperature-time graph for one day (midnight to midnight) in Durban

TRANSFORMATION GEOMETRY

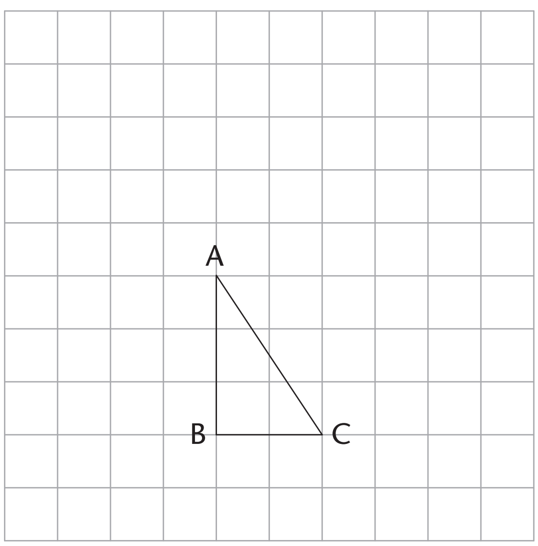
1. (a) Make tick marks in the relevant boxes in this table, to show which transformations will produce *congruent* figures:

	Translation	Reflection	Rotation	Enlargement
Congruent figures				

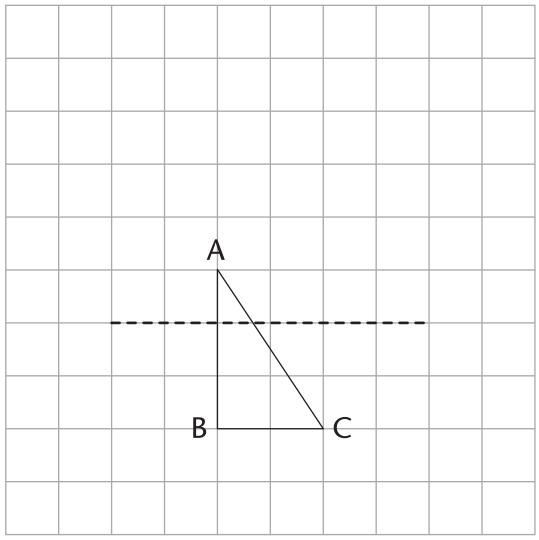
   (b) Which of the transformations will produce similar figures that are not congruent?

....

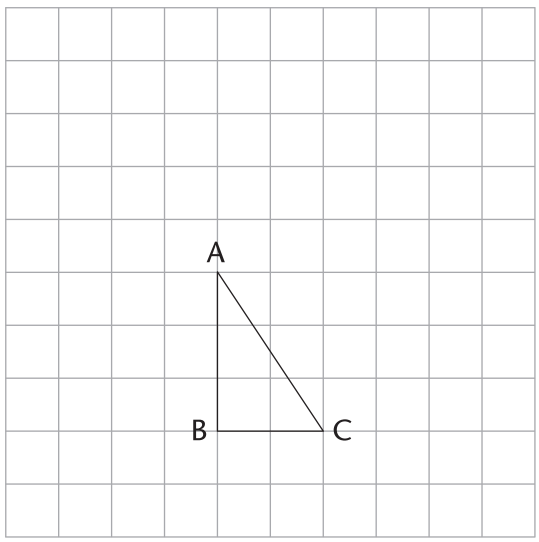
2. Perform the described transformation on shape ABC on each of the following grids, and label the image vertices A', B' and C':
   (a) Translation of 2 units to the left and 1 unit down



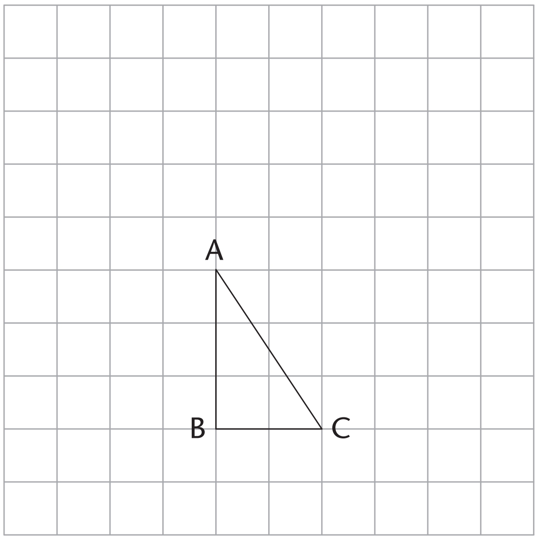
(b) Reflection in the dotted line



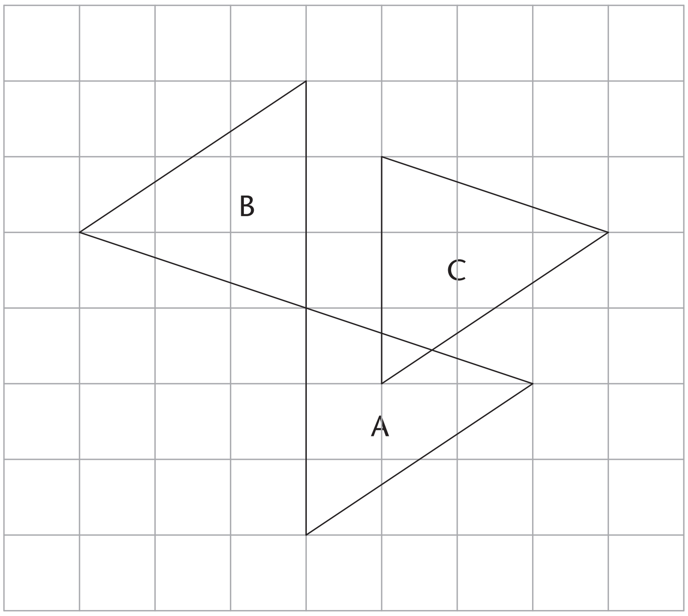
(c) Rotation of 90° clockwise around vertex A



(d) Enlargement of factor 2, with vertex B as the centre of enlargement



3. Describe in as much detail as possible the single transformation which maps triangle A onto:
   (a) triangle B
   (b) triangle C.



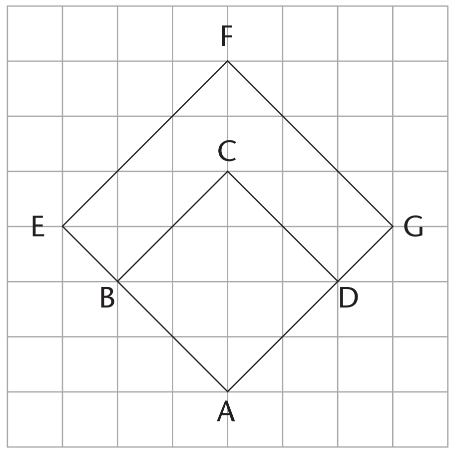
....

....

....

....

4. In this image, quadrilateral ABCD has been enlarged to quadrilateral AEFG:



(a) Write down the factor of enlargement. ....

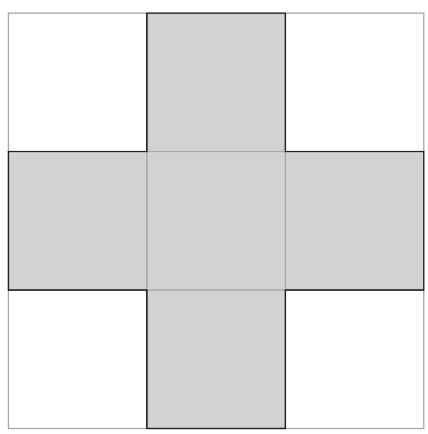
(b) What kind of quadrilateral is ABCD? ....

(c) What kind of quadrilateral is AEFG? ....

(d) Are the two quadrilaterals (ABCD and AEFG) congruent, or are they similar, or both?

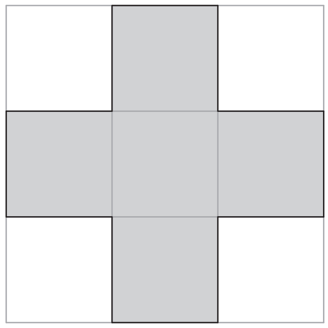
....

5. Consider the square grid below, with some blocks shaded in.

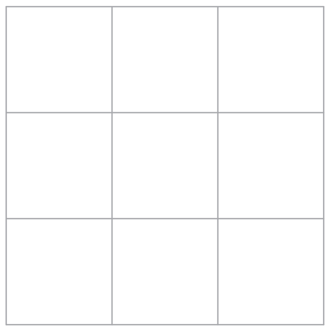


(a) How many lines of symmetry does the shape have? ....
Draw them in on the diagram, using dashed lines.

(b) Shade in some more blocks on the grid below so that the shape has exactly two lines of symmetry.



(c) Shade in some blocks on the grid below so that the shape has exactly one line of symmetry.



GEOMETRY OF 3D OBJECTS

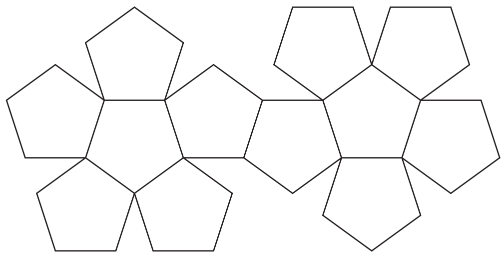
1. What is the name of the solid (3D object) that:
   (a) has only square faces? ....
   (b) has a combination of square faces and triangular faces? ....
   (c) has 3 faces and no vertices? ....
   (d) has 4 triangular faces only? ....

2. (a) Complete the table.

	Number of faces	Number of vertices	Number of edges
Rectangular prism			
Triangular prism			

(b) Euler was a Swiss mathematician who lived in the 18th century. He noticed that the number of faces (*F*) plus the number of vertices (*V*) minus the number of edges (*E*) is almost always equal to 2 for solids. Thus: $F + V - E = 2$.
Check whether Euler's formula works for the two prisms in question (a). Show all your working.

3. Study the image below, showing the net of a dodecahedron:



Write down the number of faces and vertices a dodecahedron has.

4. On the grid below, draw a possible net for a rectangular prism that is 4 blocks long, 3 blocks wide and 2 blocks high.

Assessment

In this section, the numbers in brackets at the end of a question indicate the number of marks that the question is worth. Use this information to help you determine how much working is needed.

The total number of marks allocated to the assessment is 50.

1. For each sequence, (i) describe in words the relationship between the terms in the sequence, and (ii) use the relationship to find the next three terms in the sequence:
   (a) 28,3; 31,1; 33,9; ... (2)

   ....

   ....

   (b) $\frac{2}{5}$; $\frac{6}{5}$; $\frac{18}{5}$ (2)

   ....

   ....

2. (a) Complete the table. (2)

Term number	1	2	3	4	7	
Value of the term	0	3	8	15		143

   (b) Describe in words the rule by which you could find any term in the sequence shown in the table. (1)

   ....

3. (a) If $a = b^3 - (2 \times c^2 + 8)$, what is the value of a if $b = 5$ and $c = 4$? (3)

   ....

   ....

   (b) Use the formula $p = \frac{4 \times s - a^2}{t}$ to calculate the value of t if $p = 36$, $s = 45$ and $a = 6$. (2)

   ....

   ....

   ....

4. A soccer ball costs x rand. Write down in terms of x the costs of the following items:
   (a) five soccer balls ....
   (b) a rugby ball that costs twice as much as the soccer ball ....
   (c) a dress that costs R150 more than the soccer ball ....
   (d) three soccer balls and a dress .... (4)

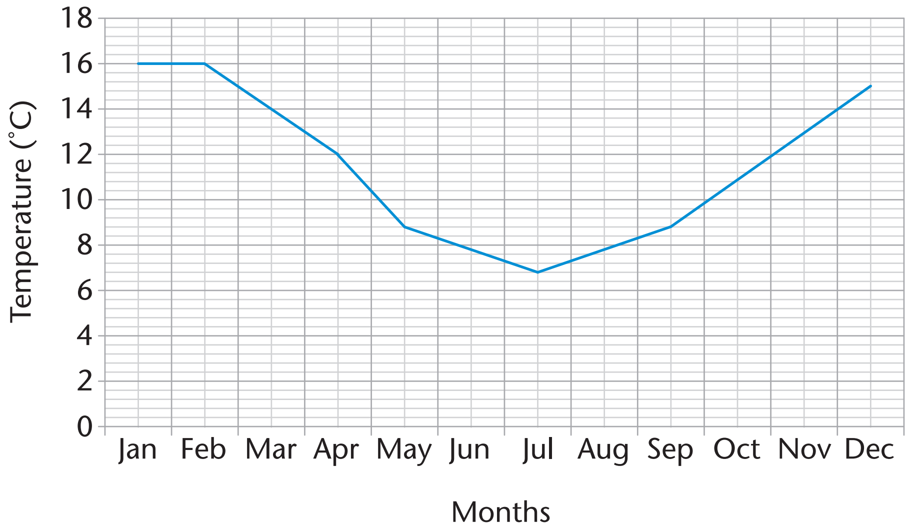
5. Solve for x:
   (a) $x + 18 = 52$ (b) $5 \times x = 60$ (c) $\frac{x}{3} = 12$ (3)

   ....

6. You are given that $2 \times x + 8 = 15$. Write down the value of $2 \times x + 10$. (1)

   ....

7. Study the following graph, showing the average minimum temperatures in Cape Town, and then answer the questions that follow:



   (a) Describe the trend in the minimum temperatures from July to December. (1)

   ....

   (b) Which month has the lowest minimum temperature, and what is the average minimum temperature in this month? (2)

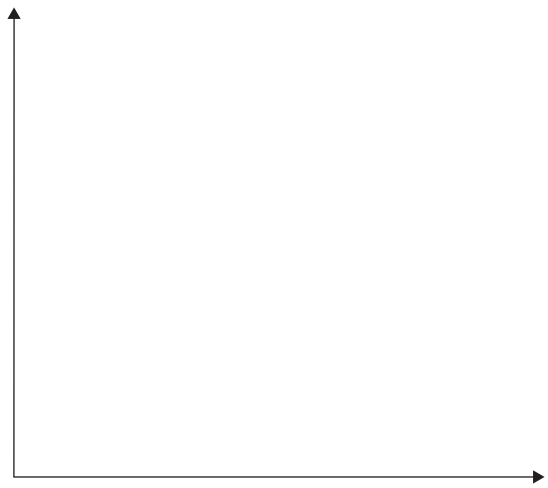
   ....

   (c) Write down the names of the months with the highest minimum temperatures, and their average minimum temperatures. (2)

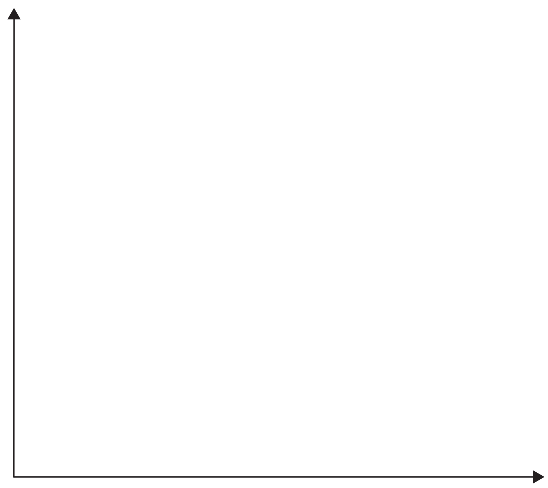
   ....

8. Draw graphs that fit each of the following descriptions. Remember to label the axes. You don't have to show any values on the axes.

(a) A graph for the total cost of different masses of stewing beef (2)

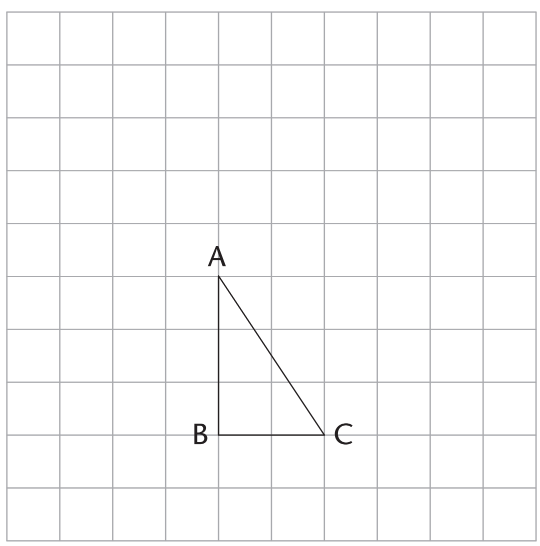


(b) A speed-time graph for a vehicle that is accelerating (2)



9. Perform the described transformation on shape ABC and label the image vertices A', B' and C':

(a) Translation by 3 units to the right and 2 units up (2)



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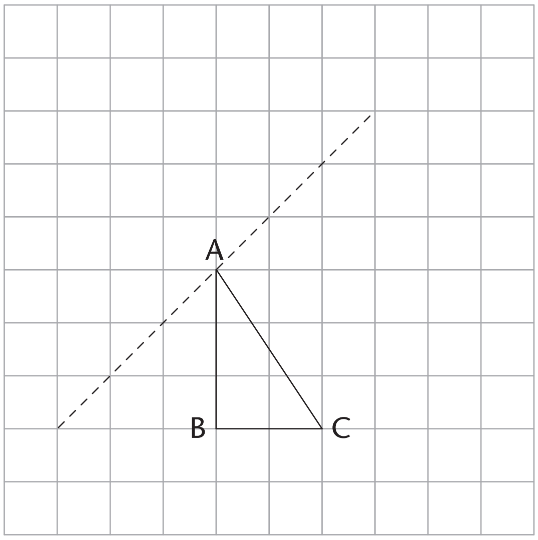
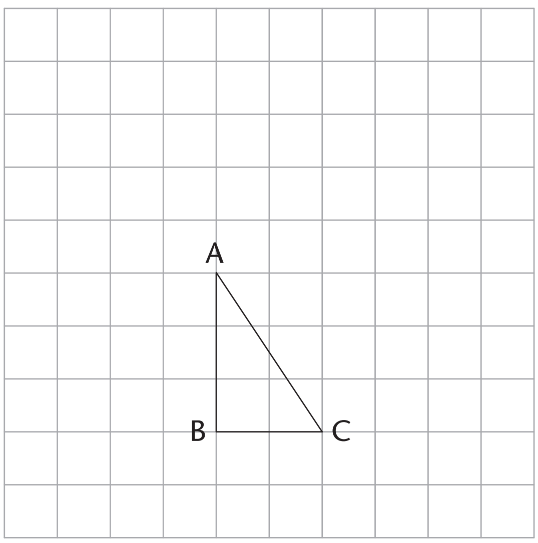
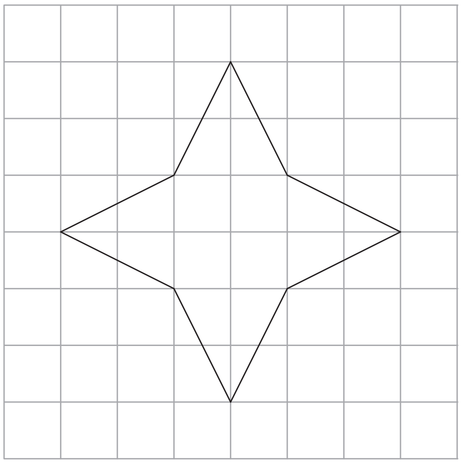
(b) Reflection in the dotted line (2)



(c) Rotation of 90° clockwise around vertex C (2)

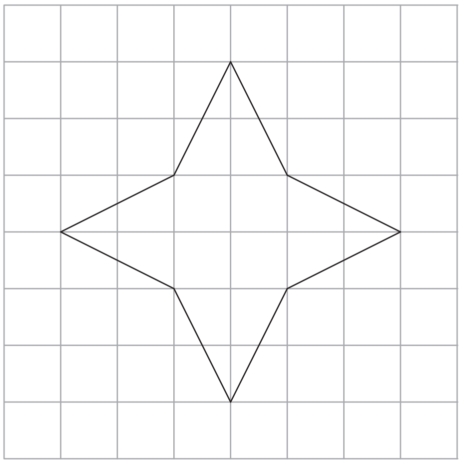


10. (a) Draw in all the lines of symmetry (use dashed lines) on the following shape. (2)

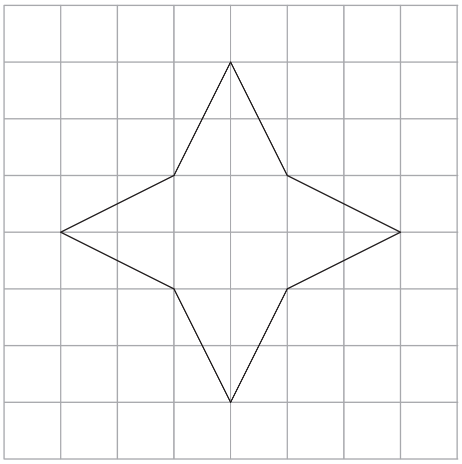


(b) Add one line from one side of the shape to the other to each of the following diagrams, so that the shape has:

(i) exactly one line of symmetry (1)



(ii) no lines of symmetry (1)



11. Complete the table. (4)

	Number of faces	**Number of vertices**	**Number of edges**
Cube			
Square-based pyramid			

12. (a) On the grid below, draw a net for a cube which has a side length of 2 units. (3)

(b) On the grid below, draw a completely different net for the same cube. It should not simply be, for example, a rotated or reflected version of the one you drew in question (a). (3)

Chapter 8
Integers

In this chapter you will work with numbers smaller than 0. These numbers are called negative numbers. Mathematicians have agreed that negative numbers should have certain properties that will make them useful for various purposes. You will learn about these properties and how they make it possible to do calculations with negative numbers.

Do what you can.

5 – 0 = ?	5 – 7 = ?	5 + 5 = ?
5 – 1 = ?	5 – 6 = ?	5 + 4 = ?
5 – 2 = ?	5 – 5 = ?	5 + 3 = ?
5 – 3 = ?	5 – 4 = ?	5 + 2 = ?
5 – 4 = ?	5 – 3 = ?	5 + 1 = ?
5 – 5 = ?	5 – 2 = ?	5 + 0 = ?
5 – 6 = ?	5 – 1 = ?	5 + ? = ?
5 – 7 = ?	5 – 0 = ?	5 + ? = ?
5 – 8 = ?	5 – ? = ?	5 + ? = ?
5 – 9 = ?	5 – ? = ?	5 + ? = ?
5 – 10 = ?	5 – ? = ?	5 + ? = ?
5 – 11 = ?	5 – ? = ?	5 + ? = ?
5 – 12 = ?	5 – ? = ?	5 + ? = ?
5 – 13 = ?	5 – ? = ?	5 + ? = ?

8 Integers

8.1 The need for numbers called integers

Numbers are used for many different purposes. We use numbers to say how many objects there are in a collection, for example the number of desks in a classroom. For this purpose we use the **counting numbers** 1, 2, 3, 4 . . . Numbers are also used to describe size, for example the lengths of objects. For this purpose we need more than the counting numbers, we also need **fractions**. Another purpose of numbers is to indicate position, for example the position of the right end of the red line on the pictures below.

Numbers also occur as the solutions to equations, and the natural numbers and fractions do not provide solutions for all equations. For example, there is no natural number or fraction that is the solution to the equation $10 - x = 20$. The number that provides the solution to this equation must have the property that when you subtract it, it has the same effect as when you add 10!

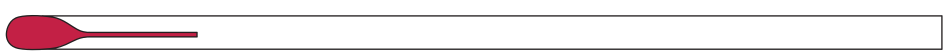
With a view to have numbers that can serve more purposes than counting and measuring, mathematicians have decided to also think of another kind of numbers which are called **integers**. The integers include the natural numbers, but for each natural number, for example 24, there is also another number called the **additive inverse**. For example, –24 is the additive inverse of 24. When you add a number to its additive inverse, the answer is 0. For example, 24 + (–24) = 0.

SAYING HOW COLD IT IS

One of the uses of integers is for the measurement of temperature. If we say that the temperature is 0 when water freezes to become ice, we need numbers smaller than 0 to describe the temperature when it gets even colder than when water freezes. When water starts boiling, its temperature is 100 degrees on the scale called the Celsius scale.

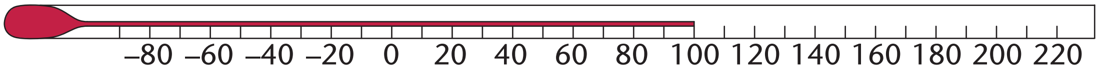
Liquids expand when heated, and shrink when cooled down. So when it is warm, the liquid in a thin tube may almost fill the tube:

When it is cold, the column of liquid will be quite short.

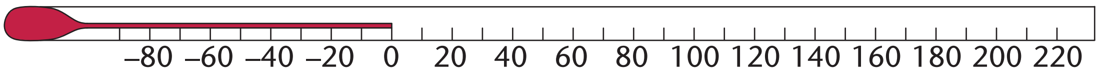


This property of liquid is used to measure temperature, and an instrument like the above is called a **thermometer**.

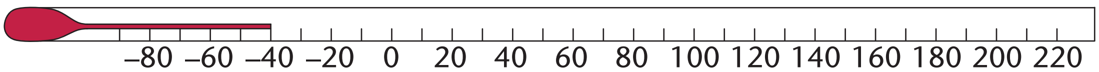
This is what a thermometer will show when it is put in water that is boiling. It shows a temperature of 100 degrees Celsius, which is written as 100 °C.



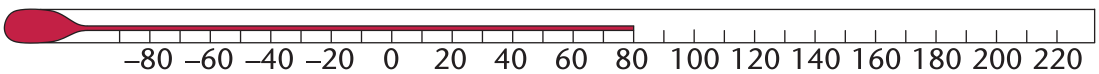
On the diagram below, you can see what a thermometer will show if it is in water that is starting to freeze. It shows a temperature of 0 °C.



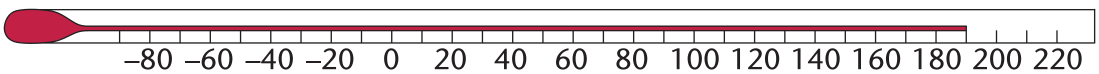
On the next diagram you can see what a thermometer will show when the temperature is –40 °C, which is colder than any winter night you may have experienced.



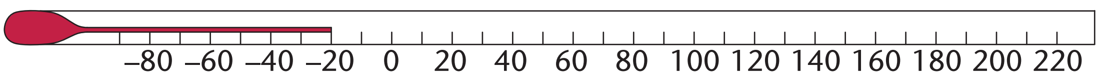
1. Write down the temperature that is shown on each of the thermometers below.



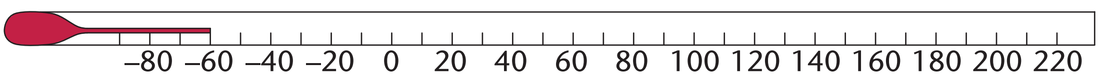
(a) ....



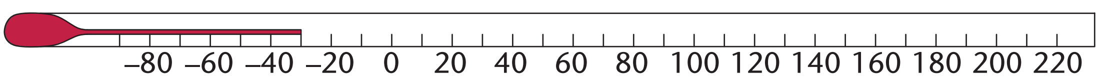
(b) ....



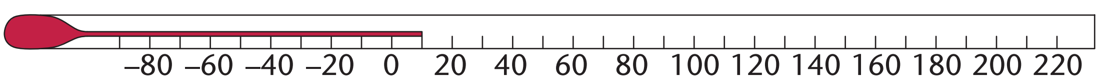
(c) ....



(d) ....



(e) ....



(f) ....

2. (a) The temperature of water in a pot is 20 °C. It is heated so that it gets 30 °C warmer. What is the temperature of the water now? ....

   (b) The temperature of water in a bottle is 80 °C. During the night it cools down to 30 °C. By how much has it cooled down? ....

   (c) In the middle of a very cold winter night the temperature outside is –20 °C. At nine o'clock in the morning it has become 30 degrees warmer. What is the temperature at nine o'clock? ....

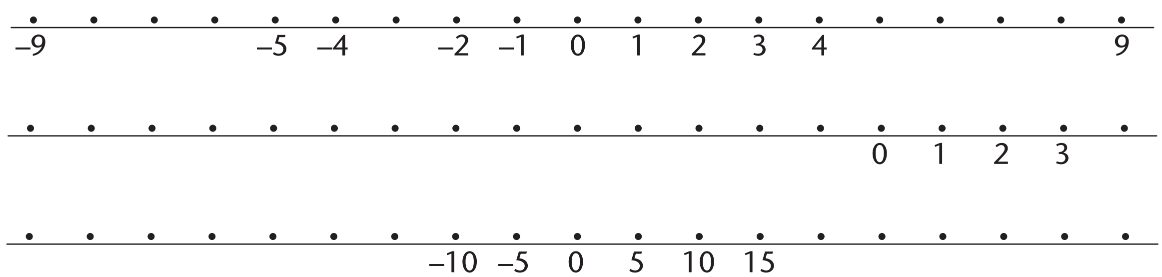
3. (a) The temperature is 8 °C. What will the temperature be if it gets 10 degrees colder? ....

   (b) The temperature is 8 °C. What will the temperature be if it gets 20 degrees colder? ....

   (c) The temperature is –8 °C. What will the temperature be if it gets 10 degrees warmer? ....

   (d) The temperature is –24 °C. What will the temperature be if it gets 10 degrees warmer? ....

4. Some numbers are shown on the number lines below. Fill in the missing numbers.



SAYING HOW MUCH MONEY IT IS

Simon is in Grade 5. He saved money in a tin. When he turned 10, his grandmother gave him R100. He also opened his savings tin on his tenth birthday and there was R260 in the tin. Simon was very happy. He said to himself: "I am very rich!"

Simon decides to buy some things that he has always wanted. This is what he decides to buy:

- a soccer ball at R160
- a pair of sunglasses at R180
- a book about animals at R90

1. How much money did Simon have in total on the day that he thought he was rich? ....

2. What is the total cost of the three items he wants to buy?

....

3. Simon decides to first buy the soccer ball only. How much money will he have after paying for the soccer ball?

....

4. How much money will Simon have if he buys the soccer ball and the sunglasses?

....

....

5. How much money will Simon have if he buys the soccer ball and the sunglasses and the book about animals?

....

....

Simon did these calculations while he was thinking about buying the various items:

R360 – R160 = R200
R200 – R180 = R20
R20 – R90 = (–) R70 ?

6. Fatima owns a small shop. One afternoon when she closed the shop, she had R120 cash, clients owed her R90, and she owed her suppliers R310. In Fatima's view her financial position was as follows: R120 + R90 – R310 = –R100.
   (a) On another day, Fatima ended the business day with R210 cash, clients owed her R180 and she owed her suppliers R160. What was her financial position?

....

....

   (b) On another day, Fatima ended the business day with R150 cash, clients owed her R130 and she owed her suppliers R460. What was her financial position?

....

....

Mathematicians are people who do mathematics for a living. Mathematics is their profession, like health care is the profession of nurses and medical doctors.

About 500 years ago, some mathematicians proposed that a "negative number" may be used to describe the result in a situation like the above, where a number is subtracted from a number smaller than it.

For example, we may say 10 – 20 = (–10)

This proposal was soon accepted by other mathematicians, and it is now used all over the world.

7. Continue the lists of numbers below to complete the table.

(a)	(b)	(c)	(d)	(e)	(f)	(g)
10	100	3	–3	–20	150	0
9	90	6	–6	–18	125	–5
8	80	9	–9	–16	100	–10
7	70	12	–12	–14	75	–15
6	60	15	–15		50	–20
5	50					–25
4	40					
3	30					
2	20					
1	10					
0	0					
–1						

8. Calculate each of the following:

(a) 16 – 20 = ... (b) 16 – 30 = ... (c) 16 – 40 = ...

(d) 16 – 60 = ... (e) 16 – 200 = ... (f) 5 – 1 000 = ...

9. Jeminah has R200 in a savings account and R40 in her purse. Her brother owes her R50. How rich is she? In other words, how much money does she have?

....

....

10. Oops! Jeminah forgot that she borrowed R60 from her mother, and that she still has to pay R150 for a dress she bought last month. So how rich (or poor) is she really? In other words, how much money does she actually have?

....

....

11. In fact, Jeminah's financial situation is even worse. She has received an outstanding bill from her doctor, for R250. So how much money does she really have?

....

....

ORDERING AND COMPARING INTEGERS

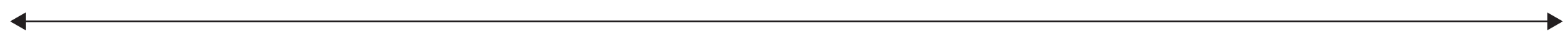
1. On a certain day the following minimum temperatures were provided by the weather bureau:

Bethlehem	–4 °C	Bloemfontein	–6 °C
Cape Town	7 °C	Dordrecht	–9 °C
Durban	12 °C	Johannesburg	0 °C
Pretoria	4 °C	Queenstown	–1 °C

Arrange the temperatures from the coldest to the warmest.

....

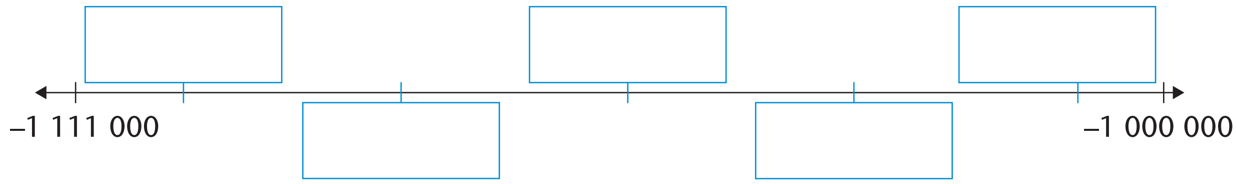
2. Place the following numbers on the number line as accurately as you can:
50; –2; –23; 5; –36



3. In each case, place the numbers in the boxes provided:

(a) 125 000; –178 000; –100 900; 180 500

(b) –1 055 500; –1 010 100; –1 100 100; –1 032 800; –1 077 500



4. Insert one of the symbols > or < to indicate which number is the smaller of the two.

(a) 978 543 ☐ 978 534 (b) –1 043 724 ☐ –1 034 724

(c) –864 026 ☐ –864 169 (d) –103 232 ☐ –104 326

(e) –710 742 ☐ 710 741 (f) –904 700 ☐ –904 704

8.2 Finding numbers that make statements true

The numbers 1, 2, 3, 4 and so on that we use for counting are called the **natural numbers**. Natural numbers are **whole numbers** – they do not contain fraction parts.

1. Is there a natural number that can be put in the brackets below to make the statement true?

   $12 + (\ldots\ldots) = 17$

2. In each case below, insert a natural number in the space between the brackets that will make the statement true.

   (a) $15 + (\ldots\ldots) = 21$

   (b) $15 - (\ldots\ldots) = 10$

   (c) $(\ldots\ldots) + 10 = 34$

   (d) $(\ldots\ldots) - 10 = 34$

   (e) $3 \times (\ldots\ldots) = 18$

> Here is a different way to ask the same questions:
> (a) What is x if $15 + x = 21$?
> (b) What is x if $15 - x = 10$?
> (c) What is x if $x + 10 = 34$?
> (d) What is x if $x - 10 = 34$?
> (e) What is x if $3 \times x = 18$?

3. (a) Can you think of a natural number that will make this statement true?

   $2 \times (\ldots) = 5$ ....

   (b) Can you think of any other number that will make the statement true?

   ....

4. (a) Can you think of a natural number that will make this statement true?

   $8 + (\ldots) = 5$ ....

   (b) Can you think of any other number that will make the statement true?

   ....

> We normally think of adding as making something bigger. Question 4(a) requires us to change our mind about this. We have to consider the possibility that adding a number may make something smaller.

We are looking for a number that will make the following statement true:

$$8 + (\ldots) = 5$$

Consider this plan:
Let us agree that we will call this number *negative 3* and write it as (–3).
If we agree to this, we can say $8 + (-3) = 5$.

This may seem a bit strange to you. You do not have to agree now. But even if you do not agree, let us explore how this plan may work for other numbers. What answers will a person who agrees to the plan give to the following question?

5. Calculate each of the following:

(a) $10 + (-3)$ .... (b) $12 + (-3)$ ....

(c) $12 + (-5)$ .... (d) $10 + (-9)$ ....

(e) $8 + (-8)$ .... (f) $1 + (-1)$ ....

What may each of the following be equal to?
$5 + (-8)$
$(-5) + (-8)$

You possibly agree that

$5 + (-5) = 0$ $10 + (-10) = 0$ and $20 + (-20) = 0$

We may say that for each "positive" number there is a **corresponding** or **opposite** negative number. Two positive and negative numbers that correspond, for example 3 and (–3), are called **additive inverses**. They wipe each other out when you add them.

When you add any number to its additive inverse, the answer is 0. For example, $120 + (-120) = 0$

So, the **set of integers** consists of all the natural numbers and their additive inverses and zero.

The number zero is regarded as an integer.

6. Write the additive inverse of each of the following numbers:

(a) 24 .... (b) –24 ....

(c) –103 .... (d) 2 348 ....

The idea of additive inverses may be used to explain why $8 + (-5)$ is equal to 3:

$$8 + (-5) = 3 + \boxed{5 + (-5)} = 3 + 0 = 3$$

7. Use the idea of additive inverses to explain why each of these statements is true:

(a) $43 + (-30) = 13$ (b) $150 + (-80) = 70$

....

....

8. Calculate each of the following:

(a) $10 + 4 + (-4)$ .... (b) $10 + (-4) + 4$ ....

(c) $3 + 8 + (-8)$ .... (d) $3 + (-8) + 8$ ....

Natural numbers can be arranged in any order to add and subtract them. It would make things easy if we agree that this should also be the case for negative numbers.

9. Calculate each of the following:

(a) $18 + 12 =$ .... (b) $12 + 18 =$ ....

(c) $2 + 4 + 6 =$ .... (d) $6 + 4 + 2 =$ .... (e) $2 + 6 + 4 =$ ....

(f) $4 + 2 + 6 =$ .... (g) $4 + 6 + 2 =$ .... (h) $6 + 2 + 4 =$ ....

(i) $6 + (-2) + 4 =$ .... (j) $4 + 6 + (-2) =$ .... (k) $4 + (-2) + 6 =$ ....

(l) $(-2) + 4 + 6 =$ .... (m) $6 + 4 + (-2) =$ .... (n) $(-2) + 6 + 4 =$ ....

10. Calculate each of the following:

(a) (–5) + 10 ....
(b) 10 + (–5) ....
(c) (–8) + 20 ....
(d) 20 – 8 ....
(e) 30 + (–10) ....
(f) 30 + (–20) ....
(g) 30 + (–30) ....
(h) 30 + (–40) ....
(i) 10 + (–5) + (–3) ....
(j) (–5) + 7 + (–3) + 5 ....
(k) (–5) + 2 + (–7) + 4 ....

11. In each case find the number that makes the statement true. Give your answer by writing a closed number sentence.

(a) 20 + (an unknown number) = 50

....

(b) 50 + (an unknown number) = 20

....

(c) 20 + (an unknown number) = 10

....

(d) (an unknown number) + (–25) = 50

....

(e) (an unknown number) + (–25) = (–50)

....

Statements like these are also called number sentences.
An incomplete number sentence, where some numbers are not known at first, is sometimes called an **open number sentence**:
8 – (a number) = 10
A **closed number sentence** is where all the numbers are known:
8 + 2 = 10

12. Use the idea of additive inverses to explain why each of the following statements is true:

(a) 43 + (–50) = –7

....
....

(b) 60 + (–85) = –25

....
....

STATEMENTS THAT ARE TRUE FOR MANY DIFFERENT NUMBERS

For how many different pairs of numbers can the following statement be true, if only natural (positive) numbers are allowed?

a number + another number = 10

For how many different pairs of numbers can the statement be true if negative numbers are also allowed?

8.3 Adding and subtracting integers

PROPERTIES OF INTEGERS

1. Calculate:

   (a) 80 + (–60) = ...

   (b) 500 + (–200) + (–200) = ...

2. (a) Do you agree that 20 + (–5) = 15? ....

   (b) What do you think 20 – (–5) should be?

   ....

3. (a) Is 100 + (–20) + (–20) = 60, or does it equal something else? ....

   (b) What do you think (–20) + (–20) should be equal to? ....

We normally think of addition and subtraction as actions that have opposite effects: what the one does is the opposite or **inverse** of what the other does.

4. Complete the following as far as you can:

(a)	(b)	(c)
5 – 9 =	5 + 9 =	9 – 3 =
5 – 8 =	5 + 8 =	8 – 3 =
5 – 7 =	5 + 7 =	7 – 3 =
5 – 6 =	5 + 6 =	6 – 3 =
5 – 5 =	5 + 5 =	5 – 3 =
5 – 4 =	5 + 4 =	4 – 3 =
5 – 3 =	5 + 3 =	3 – 3 =
5 – 2 =	5 + 2 =	2 – 3 =
5 – 1 =	5 + 1 =	1 – 3 =
5 – 0 =	5 + 0 =	0 – 3 =
5 – (–1) =	5 + (–1) =	(–1) – 3 =
5 – (–2) =	5 + (–2) =	(–2) – 3 =
5 – (–3) =	5 + (–3) =	(–3) – 3 =
5 – (–4) =	5 + (–4) =	(–4) – 3 =
5 – (–5) =	5 + (–5) =	(–5) – 3 =

5. Calculate each of the following:

(a) 20 – 20 = ...

(b) 50 – 20 = ...

(c) (–20) – (–20) = ...

(d) (–50) – (–20) = ...

6. In each case suggest a number that may make the statement true. Also give an argument to support your proposal.

(a) 20 + (a number) = 8

....

....

(b) 20 + (a number) = 28

....

....

(c) 20 – (a number) = 28

....

....

(d) 20 – (a number) = 12

....

....

SOME HISTORY

The following statement is true if the number is 5:

15 – (a certain number) = 10

A few centuries ago, some mathematicians decided they wanted to have numbers that will also make sentences like the following true:

15 + (a certain number) = 10

But to go from 15 to 10 you have to **subtract 5**.

The number we need to make the sentence *15 + (a certain number) = 10* true must have the following strange property:

If you **add** this number, it should have the **same effect** as to **subtract 5**.

Now the mathematicians of a few centuries ago really wanted to have numbers for which such strange sentences would be true. So they thought:

"Let us just decide, and agree amongst ourselves, that the number we call *negative 5* will have the property that if you add it to another number, the effect will be the same as when you subtract the natural number 5."

This means that the mathematicians agreed that 15 + (–5) is equal to 15 – 5.

Stated differently, instead of adding *negative 5* to a number, you may subtract 5.

We may agree that subtracting a negative number has the same effect as adding the additive inverse of the negative number. If we stick to this agreement, the following two calculations should have the same answer:

$$10 - (-7) \quad \text{and} \quad 10 + 7$$

7. Calculate.

(a) 20 – (–10) = ...

(b) 100 – (–100) = ...

(c) 20 + (–10) = ...

(d) 100 + (–100) = ...

(e) (–20) – (–10) = ...

(f) (–100) – (–100) = ...

(g) (–20) + (–10) = ...

(h) (–100) + (–100) = ...

8. Complete the following as far as you can:

(a)	(b)	(c)
5 – (–9) =	(–5) + 9 =	9 – (–3) =
5 – (–8) =	(–5) + 8 =	8 – (–3) =
5 – (–7) =	(–5) + 7 =	7 – (–3) =
5 – (–6) =	(–5) + 6 =	6 – (–3) =
5 – (–5) =	(–5) + 5 =	5 – (–3) =
5 – (–4) =	(–5) + 4 =	4 – (–3) =
5 – (–3) =	(–5) + 3 =	3 – (–3) =
5 – (–2) =	(–5) + 2 =	2 – (–3) =
5 – (–1) =	(–5) + 1 =	1 – (–3) =
5 – 0 =	(–5) + 0 =	0 – (–3) =
5 – 1 =	(–5) + (–1) =	(–1) – (–3) =
5 – 2 =	(–5) + (–2) =	(–2) – (–3) =
5 – 3 =	(–5) + (–3) =	(–3) – (–3) =
5 – 4 =	(–5) + (–4) =	(–4) – (–3) =
5 – 5 =	(–5) + (–5) =	(–5) – (–3) =

9. In each case, state whether the statement is true or false and give a numerical example to demonstrate your answer.
   (a) Subtracting a positive number from a negative number has the same effect as adding the additive inverse of the positive number.

   ....

   (b) Adding a negative number to a positive number has the same effect as adding the additive inverse of the negative number.

   ....

   (c) Subtracting a negative number from a positive number has the same effect as subtracting the additive inverse of the negative number.

   ....

   (d) Adding a negative number to a positive number has the same effect as subtracting the additive inverse of the negative number.

   ....

   (e) Adding a positive number to a negative number has the same effect as adding the additive inverse of the positive number.

   ....

   (f) Adding a positive number to a negative number has the same effect as subtracting the additive inverse of the positive number.

   ....

   (g) Subtracting a positive number from a negative number has the same effect as subtracting the additive inverse of the positive number.

   ....

   (h) Subtracting a negative number from a positive number has the same effect as adding the additive inverse of the negative number.

   ....

PROPERTIES OF OPERATIONS

1. Calculate the following:
   (a) $(-3) + (-5) =$ ....
   (b) $(-5) + (-3) =$ ....
   (c) $5 + (-7) =$ ....
   (d) $(-7) + 5 =$ ....
   (e) $(-13) + 17 =$ ....
   (f) $17 + (-13) =$ ....
   (g) $15 + 19 =$ ....
   (h) $19 + 15 =$ ....
   (i) $(-21) + (-15) =$ ....
   (j) $(-15) + (-21) =$ ....

In chapter 1 of Book 1 (which was about whole numbers) we said:

Addition is commutative: the numbers can be swopped around.

Or, in symbols: $a + b = b + a$, where a and b are whole numbers.

2. (a) Would you say addition is also commutative when the numbers are integers?

   ....

   (b) Explain your answer.

   ....

   ....

3. Calculate the following:

   (a) $9 - 5 =$ .... (b) $5 - 9 =$ ....

   (c) $(-7) - 3 =$ .... (d) $3 - (-7) =$ ....

   (e) $15 - (-12) =$ .... (f) $(-12) - 15 =$ ....

   (g) $(-40) - (-23) =$ .... (h) $(-23) - (-40) =$ ....

4. (a) Do you think subtraction is commutative?

   ....

   (b) Explain your answer.

   ....

   ....

In Book 1, chapter 1 we also said:

When three or more whole numbers are added, the order in which you perform the calculations makes no difference. We say: **Addition is associative**.

5. Do you think addition is also associative when we work with integers? Investigate.

   ....

   ....

   ....

   ....

Chapter 9
Numeric patterns

In this chapter you will analyse, extend and form number patterns with integers, including negative numbers.

–100	–91	–82	–73	–64	–55	–46	–37
–100	–92	–84	–76	–68	–60	–52	–44
–100	–93	–86	–79	–72	–65	–58	–51
–100	–94	–88	–82	–76	–70	–64	–58
–100	–95	–90	–85	–80	–75	–70	–65
–100	–96	–92	–88	–84	–80	–76	–72
–100	–97	–94	–91	–88	–85	–82	–79
–100	–98	–96	–94	–92	–90	–88	–86
–100	–99	–98	–97	–96	–95	–94	–93
–100	–100	–100	–100	–100	–100	–100	–100
–100	–101	–102	–103	–104	–105	–106	–107
–100	–102	–104	–106	–108	–110	–112	–114
–100	–103	–106	–109	–112	–115	–118	–121
–100	–104	–108	–112	–116	–120	–124	–128
–100	–105	–110	–115	–120	–125	–130	–135
–100	–106	–112	–118	–124	–130	–136	–142
–100	–107	–114	–121	–128	–135	–142	–149
–100	–108	–116	–124	–132	–140	–148	–156

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9 Numeric patterns

9.1 Investigating and extending numeric patterns

PATTERNS IN TWO DIRECTIONS

1. The numbers in each row of the table form a sequence, but not all the numbers are given.

A						4	6	8	10					
B						10	8	6	4					
C						5	8	11	14					
D						20	17	13	8					

(a) Fill in the missing numbers.

(b) What is the constant difference in sequence A? ....

(c) What is the constant difference in sequence C? ....

2. The first term of a certain sequence is 100 and the constant difference is 20.

(a) What is the second term, and the third term, and the fourth term?

....

(b) What is the 10th term in this sequence?

....

A constant-difference sequence is formed by adding the constant difference each time to form the next term.

3. The first term of a certain sequence is 100 and the constant difference is –20.

(a) What is the second term, and the third term, and the fourth term?

....

(b) What is the 10th term in this sequence?

....

4. (a) What is the constant difference in sequence B in question1?

....

(b) What is the constant difference in sequence D in question1?

....

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5. The sixth terms of sequences E, F and G are given in the table. Fill in the other terms.

Term number	1	2	3	4	5	6	7	8	9
E with constant difference 10						30			
F with constant difference –5						30			
G with constant difference –10						30			

6. Investigate each of the patterns below. Find the pattern and write the next four terms in the sequence.

(a) 1 4 9 16 25 ....

(b) 3 6 11 18 27 ....

(c) 20 19 17 14 10 ....

(d) 20 25 29 32 34 ....

7. Make some numeric patterns of your own.

(a)

....

(b)

....

(c)

....

(d)

....

(e)

....

(f)

....

(g)

....

(h)

....

9.2 Making patterns from rules

1. (a) Start at 30. Add –5 and write the answer. Add –5 again and write the answer. Continue until you have a number sequence with 10 terms.

   ....

   (b) Start at –30. Add –5 and write the answer. Add –5 again and write the answer. Continue until you have a number sequence with 10 terms.

   ....

   (c) Start at –30. Add 5 and write the answer. Add 5 again and write the answer. Continue until you have a number sequence with 10 terms.

   ....

2. (a) The first term of a sequence is –10 and there is a constant difference of 5 between the terms. Write down the first ten terms of the sequence.

   ....

   (b) The first term of a sequence is –10 and there is a constant difference of –5 between the terms. Write down the first ten terms of the sequence.

   ....

3. Choose a number to be your first term and another number to be a constant difference. Write the first ten terms of your sequence.

   ....

4. Choose a number smaller than –10 to be your first term and another number to be a constant difference. Write the first ten terms of your sequence.

   ....

5. Choose a number to be your first term and a negative number to be a constant difference. Write the first ten terms of your sequence.

   ....

6. Choose a negative number to be your first term and another negative number to be a constant difference. Write the first ten terms of your sequence.

   ....

7. Choose a number to be your tenth term and another number to be a constant difference. Write the first ten terms of your sequence.

   ....

8. Choose a negative number to be your tenth term and another negative number to be a constant difference. Write the first ten terms of your sequence.

   ....

9.3 Making patterns from expressions

1. (a) Complete the table.

x	0	1	2	3	4	5	6	7	8
$2 \times x - 10$									

(b) Do the output values of $2 \times x - 10$ in the above table form a pattern with a constant difference? If they do, what is the constant difference?

....

(c) Complete the table.

x	0	1	2	3	4	5	6	7	8
$3 \times x - 20$									

(d) What is the constant difference in (c)?

....

(e) Complete the table.

x	0	1	2	3	4	5	6	7	8
$2 - 3 \times x$									

(f) What is the constant difference in (e)?

....

....

(g) Complete the table.

x	0	1	2	3	4	5	6	7	8
$1 - 2 \times x$									

(h) What is the constant difference in (g)?

....

2. Look at the pattern: –15; –19; –23; –27; –31; . . .
In this pattern, –19 is followed by –23 and –23 is followed by –27.
(a) What number in the pattern is followed by –19? ....
(b) What number in the pattern is followed by –31? ....
(c) In the pattern, –19 follows on –15 and –23 follows on –19.
What number follows on –31? ....

3. A certain pattern is formed by a common difference of 6.
   (a) What number follows on 23 in this pattern? ....
   (b) What number is followed by 23 in this pattern? ....
   (c) What number follows on 47 in this pattern? ....
   (d) What number is followed by 47 in this pattern? ....

Consider the sequence: 10 6 2 –2 –6 ... ...
In this sequence, 2 follows on 6. They are called consecutive terms.

When one number follows another in a sequence they are called **consecutive terms**.

4. Write down any two consecutive terms in the pattern formed by $2 \times x + 3$, when the input numbers are consecutive whole numbers.

   ....

5. Each of the patterns below was formed by using one of the following expressions. Establish which pattern belongs to each expression.
   (a) $2 \times x + 5$ ....
   (b) $3 \times x + 2$ ....
   (c) $4 \times x + 1$ ....
   (d) $5 \times x + 6$ ....
   (e) $6 \times x - 5$ ....
   (f) $7 \times x - 2$ ....
   (g) $1 - 4 \times x$ ....
   (h) $5 - 5 \times x$ ....
   (i) $-5 - 6 \times x$ ....

A.	6	11	16	21	26	Expression ......
B.	13	17	21	25	29	Expression ......
C.	20	23	26	29	32	Expression ......
D.	1	–3	–7	–11	–15	Expression ......
E.	31	33	35	37	39	Expression ......
F.	–20	–25	–30	–35	–40	Expression ......
G.	25	31	37	43	49	Expression ......
H.	26	33	40	47	54	Expression ......
I.	–11	–17	–23	–29	–35	Expression ......

Sequence I in question 5 is a **decreasing** sequence; the numbers become smaller as the sequence progresses:

–11 –17 –23 –29 –35

Sequence H is an **increasing** sequence; each term is bigger than the previous term:

26 33 40 47 54

6. (a) Which sequences in question 5 are increasing sequences? ....

   (b) Which sequences in question 5 are decreasing sequences? ....

7. (a) By how much does sequence A increase from one term to the next? ....

   (b) By how much does sequence B increase from one term to the next? ....

   (c) Which of the sequences in question 5 increases by the biggest amount from one term to the next, and by how much does it increase?

   ....

Sequence G increases by 6 from term to term, and sequence E increases only by 2. We may say that sequence G **increases faster** than sequence E.

8. (a) Which of the sequences in question 5 decreases fastest? ....

   (b) Which of the sequences in question 5 decreases slowest? ....

9. (a) Write 5 consecutive terms of a sequence which decreases faster than sequence D in question 5.

   ....

   (b) Write 5 consecutive terms of a sequence which increases slower than sequence B in question 5.

   ....

10. (a) Each of the expressions below can be used to produce a sequence. Which of the expressions will produce the sequence that increases fastest?

    $3 \times x + 5$ $2 \times x + 10$ $6 \times x - 1$ $20 + 3 \times x$ $4 \times x - 9$

    ....

    (b) Think of a way in which you can test your answer, and do it.

    ....

    ....

    ....

11. In each case state whether the sequence will be decreasing or increasing.

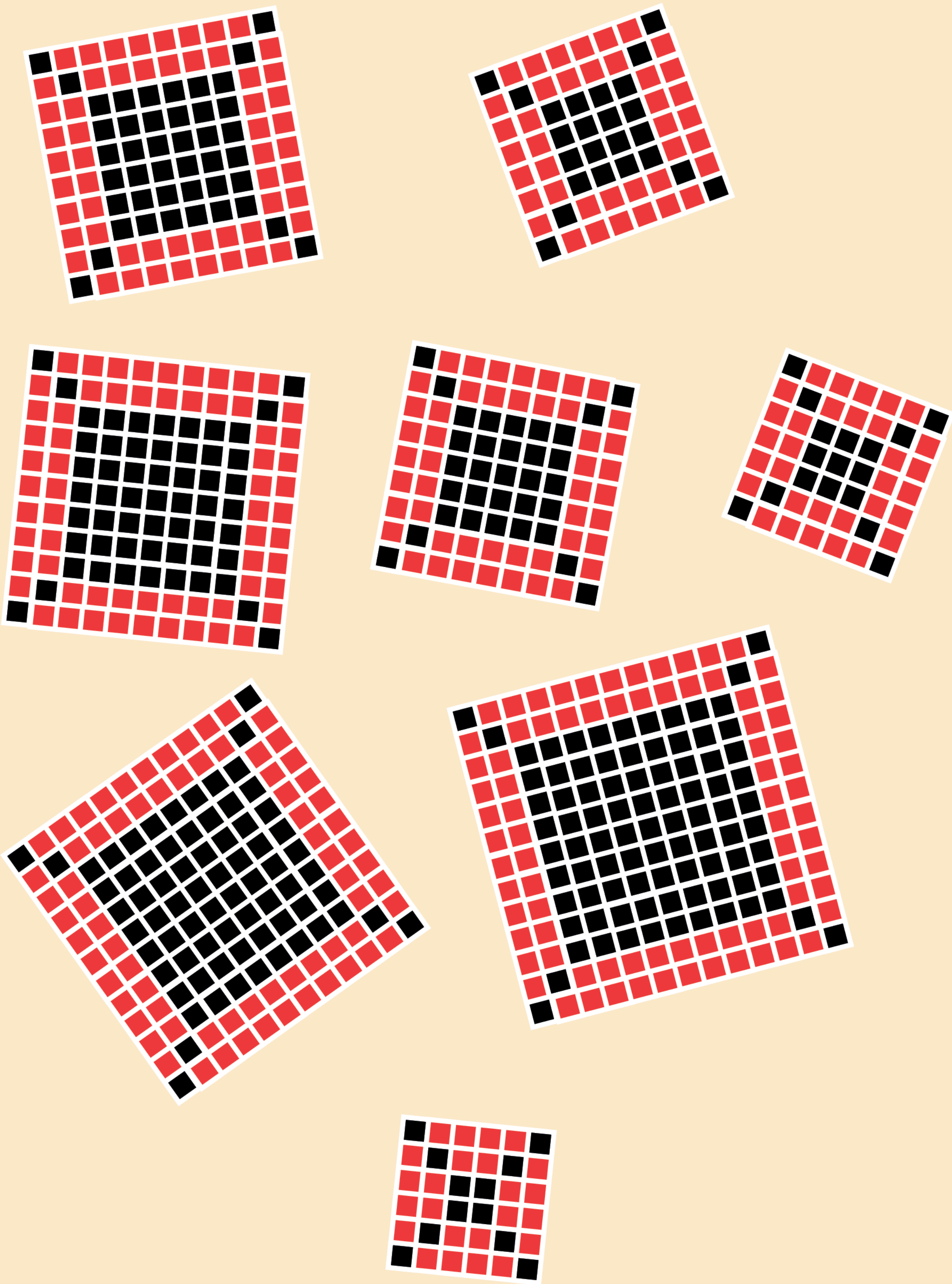
    $10 + 3 \times x$ $10 - 3 \times x$ $10 \times x + 3$ $3 \times x - 10$

    ....

Chapter 10
Functions and relationships 2

The way in which an output number can be calculated is called the rule for the relationship. The rule can be described in words or with a formula, and in some cases with a flow diagram.

The work in this chapter builds on the work that you did last term, in chapter 2.



10 Functions and relationships 2

10.1 Relationships between variables

DIFFERENT WAYS TO REPRESENT THE RULE FOR A RELATIONSHIP

A relationship between two variables consists of two sets of numbers as shown in the two rows of the table below. The first row contains the **input numbers** and the second row contains the **output numbers**.

x	1	2	3	4	5	6	7	8	9
y	32	39	46	53	60	67	74	81	88

For the relationship shown in the table, any output number can be calculated by multiplying the input number by 7 and adding 25 to the answer.

The way in which an output number can be calculated is called the **rule** for the relationship. The rule can be described in **words** or with a **formula**, and in some cases with a **flow diagram**.

The input numbers may also be called the values of the **input variable**, and the output numbers may also be called the values of the **output variable**.

The rule "multiply by 7 and add 25" can be represented with this flow diagram:

→ [× 7] → [+ 25] →

The same rule can also be represented with the formula below:

$y = 7 \times x + 25$

1. Calculate the value of $7 \times x + 25$ for each of the following values of x:

(a) $x = 10$ ....

(b) $x = 20$ ....

(c) $x = 5$ ....

(d) $x = 15$ ....

2. (a) What is the value of $3 \times x - 5$ if $x = 10$? ....

(b) What is the value of $3 \times x - 5$ if $x = 20$? ....

(c) What is the value of $3 \times x - 5$ if $x = 25$? ....

(d) What is the value of $3 \times x - 5$ if $x = 100$? ....

3. Complete the table for the values of x and $3 \times x - 5$ given in the table.

x	0	1	2	5	15		50	200	
$3 \times x - 5$						61		595	994

4. When you worked out the input number that corresponds to the output number 994 in question 3, you solved the **equation** $3 \times x - 5 = 994$.
   Write the equation that you solved when you worked out the input number that corresponds to the output number 61.

   ....

5. (a) Express each of the rules below in words.

   A → $\times 5$ → $+ 3$ → ....

   B → $- 5$ → $\times 3$ → ....

   C → $\times 3$ → $- 5$ → ....

   (b) Which of the above flow diagrams represent the same calculations as the expression $3 \times x - 5$?

   ....

Instead of $3 \times x - 5$ we may write $3x - 5$.
$3x$ means $3 \times x$.
The multiplication sign can be left out.
Instead of $3 \times (x - 5)$ we may write $3(x - 5)$.

6. (a) Which of the formulae below provide the same information as flow diagram B in question 5?

   $y = 5x - 3$ $\quad$ $y = 3 + 5x$ $\quad$ $y = 5(x - 3)$
   $y = 3x - 5$ $\quad$ $y = 5x + 3$ $\quad$ $y = 3(x - 5)$

   ....

   (b) Which of the above formulae provide the same information as flow diagram A in question 5?

   ....

FORMULAE FOR TABLES

1. The table below shows the values of y that correspond to some of the given values of x. In this case, the output numbers form a pattern with a constant difference if the input numbers are the natural numbers.

x	1	2	3	4	5	6	7	8
y	13	21		37	45	53		

(a) Find the output numbers that correspond to the input numbers 3, 7 and 8.

....

....

....

(b) Find the output numbers that correspond to the input numbers 20, 21 and 22.

....

....

....

(c) Which of the formulae below is the rule for the relationship between x and y in the above table?

$y = 10x + 3$ $y = 8x + 5$ $y = 6x + 7$ $y = 4x + 9$ $y = 2x + 11$

....

2. Complete the tables below for the formulae in question 1(c).

x	1	2	3	4	5	6	7	8
$10x + 3$								

x	1	2	3	4	5	6	7	8
$8x + 5$								

x	1	2	3	4	5	6	7	8
$6x + 7$								

x	1	2	3	4	5	6	7	8
$4x + 9$								

x	1	2	3	4	5	6	7	8
$2x + 11$								

3. In each table in question 2, the output numbers form a number pattern with a constant difference between consecutive terms. What is the constant difference in the pattern generated by each of the following expressions, when the input numbers are consecutive natural numbers? Write your answers in the table below. Also fill in the values of the expressions for $x = 0$ in the last column.

Expression	Constant difference between output numbers	Value of the expression for $x = 0$
$2x + 11$		
$4x + 9$		
$6x + 7$		
$8x + 5$		
$10x + 3$		

4. What do you think the constant differences between consecutive output numbers, and the values of the expressions for $x = 0$ may be in each of the following cases, when the input numbers are consecutive natural numbers?

Expression	Constant difference between output numbers	Value of the expression for $x = 0$
$5x + 7$		
$3x + 10$		
$12x + 5$		
$5x - 5$		
$(-10x) + 3$		

10.2 Integers in the rules for relationships

RULES THAT MAY LOOK STRANGE AT FIRST

1. Complete the flow diagrams.

A. 5 → $\times 3$ → -5 →

B. 5 → $\times 3$ → $+(-5)$ →

C. 5 → -3 → $\times 5$ →

D. 5 → $+(-3)$ → $\times 5$ →

E. 5 → $\times 3$ → $-(-5)$ →

F. 5 → $\times 3$ → $+5$ →

2. Describe each rule in question 1 in words, for example "*multiply by 6 and then add –3*".

   A. ....

   B. ....

   C. ....

   D. ....

   E. ....

   F. ....

The rule *multiply by 6 and subtract the answer from 100* can be expressed with the formula $y = 100 - 6x$. This formula can also be written as $y = 100 + (-6x)$ or as $y = (-6x) + 100$.

The brackets around the $-6x$ can be left out, so the last formula above can also be written as $y = -6x + 100$.

3. Calculate y if $y = -10x + 3$, for each of the following values of x:

   (a) $x = 5$

   ....

   ....

   (b) $x = 10$

   ....

   ....

   (c) $x = 20$

   ....

   ....

   (d) $x = 1$

   ....

   ....

4. Describe each of the rules in question 1 with a formula, for example $y = 5x + 8$.

   A. .... B. ....

   C. .... D. ....

   E. .... F. ....

5. In each case below, predict which of the different expressions will produce the same results. You will test your predictions later, and can then mark your own answers for this question.

   (a) $20 - 5x$ $5x - 20$ $(-5x) + 20$ $20 + (-5x)$

   ....

   (b) $20 + 5x$ $5x + 20$ $20x + 5$ $20 - (-5x)$ $5(x + 4)$

   ....

   (c) $5x - 20$ $20x - 5$ $(-20) - (-5x)$ $-((-5x) + 20)$

   ....

6. Complete the table below and then use the results to carefully check your answers to question 5.

x	0	1	5	10	100
$20 - 5x$					
$5x - 20$					
$(-5x) + 20$					
$20 + (-5x)$					
$20 + 5x$					
$5x + 20$					
$20x + 5$					
$20 - (-5x)$					
$5(x + 4)$					
$5x - 20$					
$20x - 5$					
$(-20) - (-5x)$					
$-((-5x) + 20)$					

7. In each case below, use your results in the above table or other methods to establish for which values of x the two expressions have the same value(s).

(a) $20 - 5x$ and $20 + 5x$

....

(b) $20 - 5x$ and $(-5x) + 20$

....

(c) $5x - 20$ and $(-20) - (-5x)$

....

(d) $5(x + 4)$ and $5x - 20$

....

(e) $20 + 5x$ and $20 - (-5x)$

....

Chapter 11
Algebraic expressions 2

You already know that an algebraic expression is a computational procedure. It tells you what calculations you must do with the value of one variable, to produce the value of another variable. In this chapter, we extend the work you have already done to include algebraic expressions with integer constants, including negative numbers.

$(-3) - 5 \times x =$	
45	–228
41	–208
37	–188
33	–168
29	–148
25	–128
21	–108
17	–88
13	–68
9	–48
5	–28
1	–8

11 Algebraic expressions 2

11.1 Interpret rules to calculate values of a variable

RULES IN VERBAL AND SYMBOLIC FORM

1. Do this to each of the numbers in the top row of the table, and write your answers in the bottom row: *multiply the input number by 20 and add 50 to the answer*.

x	1	2	3	4	5	6	7	8	9
y									

The sentence *multiply the input number by 20 and add 50 to the answer* is the rule that describes how the output number that corresponds to each input number in the above relationship between the variables x and y can be calculated.

The same rule can be described with the algebraic expression $20x + 50$. In this expression, the symbol x represents the input variable (the values of x). The numbers 20 and 50 are constant; they remain the same for all the different values of x.

The rule *add 50 to the input number and multiply the answer by 20* can be described with the expression $20(x + 50)$.

If there are no brackets in an expression, multiplication is done first, even if it appears later in the expression like in $30 + 5x$.

If there are brackets in an algebraic expression, the operations in brackets are to be done first.

2. Describe each of the following rules in words.

   (a) $15x + 30$ ....

   (b) $30 + 15x$ ....

   (c) $15(x + 30)$ ....

   (d) $15(x + 2)$ ....

   (e) $15x - 30$ ....

   (f) $15(x - 30)$ ....

   (g) $15(x - 2)$ ....

3. What is the difference between $3(x + 5)$ and $3x + 5$?

....

....

....

4. Complete the table.

x	1	2	3	4	5	6	7	8	9
$15x + 30$									
$30 + 15x$									
$15(x + 30)$									
$15(x + 2)$									

5. Complete the table.

x	30	40	50	60	70	80	90
$15x - 30$							
$15(x - 30)$							
$15(x - 2)$							

6. (a) Investigate which of the following rules will produce the same output numbers. You need to check for several different input numbers.

A: Multiply the input number by 10 and then add 20.

B: Add 20 to the input number and then multiply by 10.

C: Add 2 to the input number and then multiply by 10.

D: Multiply the input number by 3, add 15, add 7 times the input number, and then add 5.

x								
A								
B								
C								
D								

....

(b) Describe each of the above rules with an algebraic expression.

A: ....

B: ....

C: ....

D: ....

7. (a) Which of these rules do you think will produce the same output numbers?

A: $5x + 20$ B: $4x + 19$ C: $5(x + 20)$

D: $20 + 5x$ E: $5(x + 4)$ F: $3x + 7 + 2x + 13$

....

(b) Express each of the above rules in words.

A: ....

B: ....

C: ....

D: ....

E: ....

F: ....

....

(c) Complete this table for the rules given in (a).

x	0	5	10	15
$5x + 20$				
$4x + 19$				
$5(x + 20)$				
$20 + 5x$				
$5(x + 4)$				
$3x + 7 + 2x + 13$				

(d) Use your completed table to check your answer in question (a).

8. (a) Which of these rules do you think will produce the same output numbers?

A: $5x - 20$ B: $20 - 5x$ C: $5(x - 20)$

D: $3x - 18$ E: $5(x - 4)$ F: $9x + 10 - 4x - 30$

....

....

(b) Express each of the above rules in words.

A: ....

B: ....

C: ....

D: ....

E: ....

F: ....

....

(c) Complete this table for the rules given in (a).

x	20	30	40	50	60	70	80	90
$5x - 20$								
$20 - 5x$								
$5(x - 20)$								
$3x - 18$								
$5(x - 4)$								
$9x + 10 - 4x - 30$								

(d) Use your completed table to check your answer to question (a).

11.2 Slightly different kinds of rules

SUBTRACT POSITIVE AND NEGATIVE QUANTITIES

1. Complete the table.

x	1	10	5	20	25
$10x$					
$50 - 10x$					
$20 - 10x$					
$0 - 10x$					

2. (a) Complete the table.

x	0	5	10	15	20	25	30
$10x - 5$							
$5x - 10$							
$100 - 5x$							
$-100 + 5x$							
$5x - 100$							
$5 - 10x$							

(b) The values of $10x - 5$ *increase* as the values of x increase from 0 to 30. For which expressions in (a) do the values *decrease* when x is increased?

....

(c) Do the values of $-100 + 5x$ *increase* or *decrease* when x is increased from 0 to 30?

....

3. (a) The values of the expression $5x - 10$ increase when x is increased from 0 to 30. Do you think the values will increase further when x is increased beyond 30, or will they start to decrease at some stage? ....

(b) Do you think the values of the expression $100 - 3x$ will increase when x is increased from 0 to 30? Explain why you think they will or will not.

....

....

The additive inverse of a number may be indicated by writing a negative sign before the number. For example, the additive inverse of 8 can be written as –8.

4. Write the additive inverse of each of the following numbers:

20 30 –25 –20 40

....

When a number is added to the number called its additive inverse, the answer is 0. For example, 45 + (–45) = 0 and (–12) + 12 = 0.

5. Different values for x are given in the first row of the table below. Write the additive inverses of the x values in the second row, and then complete the table.

x	5	10	15	20	25	30
the additive inverse of x						
20 + (the additive inverse of x)						
20 – (the additive inverse of x)						
$20 + x$						
$20 - x$						

6. Complete the table.

x	–5	–10	–15	–20	–25	–30
the additive inverse of x						
20 + (the additive inverse of x)						
20 – (the additive inverse of x)						
$20 + x$						
$20 - x$						

7. Complete the table.

x	3	2	1	0	–1	–2	–3
$-x$							
$5 + (-x)$							
$5 - (-x)$							
$5 - x$							
$5 + x$							

EXPRESSIONS WITH ADDITIVE INVERSES

1. Complete the table.

x	1	5	10	20	25
$5x$					
the additive inverse of $5x$					
20 + (the additive inverse of $5x$)					
20 – (the additive inverse of $5x$)					
$3x$					
$-3x$					
$10 + (-3x)$					
$10 - 3x$					
$10 - (-3x)$					

2. Complete the table below.
   Note that $(-10x)$ indicates the additive inverse of $10x$.

x	1	2	3	4	–4	–3	–2
$10x - 1\ 000$							
$1\ 000 - (-10x)$							
$1\ 000 - 10x$							
$(-10x) + 1\ 000$							
$10x + 1\ 000$							
$10x + (-1\ 000)$							
$(-10x) - 1\ 000$							
$1\ 000 + (-10x)$							
$1\ 000 + 10x$							
$10x - (+1\ 000)$							

Instead of $(-10x) - 1\ 000$ we may write $-10x - 1\ 000$, in other words the brackets around the additive inverse may be left out.

Similarly, $(-10x) + 1\ 000$ may be written as $-10x + 1\ 000$.

3. Complete the table.

x	1	5	10	20	25	30
$-5x + 20$						
$-5x + (-20)$						

Chapter 12
Algebraic equations 2

You have already done some work on equations in Term 3. In this term, we extend the work you have already done to include negative numbers.

5	×	21	– 3	=	102
5	×	?	– 3	=	97
5	×	?	– 3	=	92
5	×	?	– 3	=	87
5	×	?	– 3	=	82
5	×	x	– 3	=	77
5	×	?	– 3	=	72
5	×	?	– 3	=	67
5	×	?	– 3	=	62
5	×	?	– 3	=	57
5	×	?	– 3	=	52
5	×	?	– 3	=	47

12 Algebraic equations 2

12.1 Describing problem situations

A **closed number sentence** is a true statement about numbers, for example $21 + 5 = 26$. All the numbers are given.

In an **open number sentence**, for example $15 + x = 21$, one or more of the numbers are unknown.

An open number sentence is also called an **equation**.

1. Jan is 3 years older than his sister Amanda. Amanda is 14 years old. Write a closed number sentence to show Jan's age.

   ....

2. Numbers are said to be consecutive if they follow one another. The numbers $-1,\ 0,\ 1$ are consecutive. The sum of $-1,\ 0$ and 1 is 0.
   (a) Write a closed number sentence that shows two consecutive numbers that add up to -33.

   ....

   (b) Write a closed number sentence that shows two consecutive numbers whose product is 6.

   ....

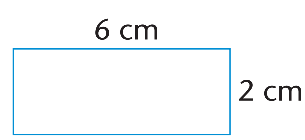
3. A cell phone costs R500 after a discount of R150 is given. Write a closed number sentence to show the original price of the cell phone.

   ....

4. When the bus leaves the terminal, it is carrying 55 people. At the first bus stop 12 people get off the bus and 9 people get in. At the second bus stop, 12 people get in and 9 people get off the bus. Write a closed number sentence to show the number of people that are now in the bus.

   ....

5. A rectangle is shown on the right. Write a closed number sentence to calculate the following:



   (a) the area of the rectangle ....
   (b) the perimeter of the rectangle ....

12.2 Analysing and interpreting equations

1. The cost of a school uniform in rands is represented by x. An alteration fee of R20 is also charged. Mr Malan paid R520 for both the school uniform and the alterations done on it.
   (a) Which equation describes this situation?
   A. $20 \times x = 520$ B. $x - 20 = 520$ C. $x + 20 = 520$ D. $20 + 20 = x$

   (b) What is the price of the uniform?

2. Five learners should each receive the same number of sweets. There are 60 sweets in total that they have to share.
   (a) Which equation describes this situation?
   A. $5 + s = 60$ B. $5s = 60$ C. $s - 5 = 60$ D. $\frac{s}{5} = 60$

   (b) How many sweets does each learner get?

   (c) What does the letter s represent in the equation you have chosen?

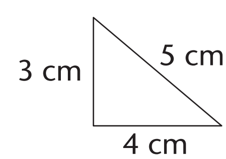
3. A taxi picks up n passengers at the airport and drives to the nearest hotel. When it leaves the hotel, the number of passengers in the taxi has decreased by 6. There are now 7 passengers in the taxi.
   (a) Which equation describes this situation?
   A. $n - 6 = 7$ B. $7 - n = 6$ C. $n + 6 = 7$ D. $n - 7 = 6$

   (b) How many passengers were in the taxi when it left the airport?

4. Write a closed number sentence for calculating the perimeter of an equilateral triangle whose sides are 5 cm long.

Remember: An equilateral triangle is a triangle in which all three sides are equal.

5. Write a closed number sentence to calculate the perimeter of the triangle shown on the right.



12.3 Solving and completing equations

SOLVE BY INSPECTION

1. The number sentences given below are not true. Make the number sentences true by changing the numbers in blue.

(a) $13 + 7 = 22$ (b) $50 + (-50) = -100$ (c) $7 \times 8 = 54$

....

(d) $9 - (-3) = 6$ (e) $-5 + 12 = -7$ (f) $4 \times 6 = 28$

....

(g) $6 - 9 = 3$ (h) $9 - 6 = -3$ (i) $5 + (-12) = 7$

....

(j) $10 + (-2) = 12$ (k) $(-1) - (-1) = -2$ (l) $0 + (-2) = 0$

....

2. Consider the equations given below. Check whether the value given in brackets is the solution. Simply write *yes* or *no* with an explanation.

(a) $x + 3 = 0$ $(x = -3)$

....

(b) $3 - x = 4$ $(x = 1)$

....

(c) $-5 + x + x = -11$ $(x = -2)$

....

(d) $3 - x = 4$ $(x = -1)$

....

To check whether a given value is the solution or not we have to answer the following question in our minds: **Does the given value make the equation true?** If it does, we say such a value is the **solution**.

3. Find the value of the unknown that makes the equation true in each case:

(a) $x + 6 = 8$ (b) $x + 6 = 4$ (c) $x + 6 = 0$

....

(d) $6 - x = 8$ (e) $6 - x = 4$ (f) $6 - x = 0$

....

(g) $\frac{x}{4} = 2$ (h) $x = 4 \times 2$ (i) $\frac{x}{2} = \frac{1}{4}$

....

4. Three possible solutions are given in brackets below each equation, but only one is correct. Find the correct solution in each case.

(a) $x + 27 = 27$
{–27; 0; 1}

....

(b) $12 = 4 - x$
{8; 16; –8}

....

(c) $x + 3 = 0$
{–3; 0; 3}

....

(d) $5 - x = 10$
{–5; 0; 5}

....

(e) $5 + x = 10$
{–5; 0; 5}

....

(f) $-5 + x = 10$
{–5; –15; 15}

....

(g) $-5 - x = 10$
{–5; –15; 15}

....

(h) $-5 - x = 0$
{–5; –15; 15}

....

(i) $5 - x = -10$
{–5; –15; 15}

....

(j) $x = \frac{10}{10}$
{0; 1; 100}

....

(k) $10x = 0$
$\{0; 1; \frac{1}{10}\}$

....

(l) $\frac{x}{10} = 0$
{0; 1; 10}

....

5. What value for x would make each equation below true?

(a) Let x = ... then $x + 3 = 10$

....

(b) Let x = ... then $x + 3 = -4$

....

(c) $x + x + x = -6$ is true for x = ...

....

(d) $x + x + x + x = -8$ is true for x = ...

....

6. In each case, fill in the table until you can see for what value of x the equation given above the table is true. You may add more x values of your own choice. To save time and work, you may skip columns that you think will not help you to find the solution.

(a) $37 - 4x = 5$

x	1	10	5	6	7				
$37 - 4x$									

(b) $50 - 7x = 22$

x	1	10	5	6					
$50 - 7x$									

(c) $100 - 3x = 49$

x	10	20	25	15	16				
$100 - 3x$									

SOLVE BY TRIAL AND IMPROVEMENT

We can think of an equation as a question asking for a value that we can assign to the **unknown** to make the equation true.

Consider the equation $82 + m = 23$. We need to assign values to m until we find a value that makes the equation true, as shown in the table below.

	Equation	True/False
Let $m = -50$	$82 + (-50) = 82 - 50 = 32$	False
Let $m = -30$	$82 + (-30) = 82 - 30 = 52$	False
Let $m = -60$	$82 + (-60) = 82 - 60 = 22$	False
Let $m = -59$	$82 + (-59) = 82 - 59 = 23$	True

So $m = -59$ because $82 + (-59) = 82 - 59 = 23$

1. Determine the value of t that makes the equation $28 - t = 82$ true by making use of the trial and improvement method.

	Equation	True/False

Solution: ....

2. Consider the equation $w + 32 = -68$. Use the trial and improvement method to find the solution of the equation.

	Equation	True/False

Solution: ....

3. The equation $200 - 5t = 110$ is given. What value of t makes the equation true? Use the table below to determine the solution.

	Equation	**True/False**

Solution: ....

4. What value of p makes the equation $18p = 90$ true?

	Equation	**True/False**

Solution: ....

5. What value of x makes the equation $88 - 6x = 46$ true?

	Equation	**True/False**

Solution: ....

12.4 Identifying variables and constants

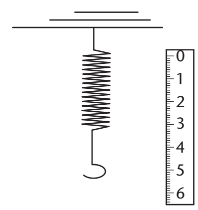
1. The mass of an empty truck is 2 680 kg. The truck is used to transport cement. Each pocket of cement has a mass of 90 kg.

   The combined mass of the truck and the cement can be calculated by means of the formula: $y = 90 \times x + 2\,680$.

   Use the terms **variable** or **constant** to describe the meaning of each symbol used in the formula. Explain your answer.

   (a) y (b) 90 (c) x (d) 2 680

2. A steel spring is suspended from a stand. Mass pieces of equal mass are hooked onto the bottom end of the spring. The length of the spring is measured with 1 mass piece hooked, 2 mass pieces hooked, 3 mass pieces hooked and so on. The results are shown in the table below.



Number of mass pieces	1	2	3	4	5	7	10
Length of spring in cm	48	56	64	72	80	96	120

The formula $y = 8x + 40$ is used to predict the length of the spring for the various number of mass pieces hooked.

Use the terms **variable** or **constant** to describe each symbol used in the formula. Explain your answer.

(a) y (b) 8 (c) x (d) 40

12.5 Numerical values of expressions

SUBSTITUTING NUMBERS INTO EXPRESSIONS

1. (a) Calculate the values of each expression for the given values of x, and write your answers in the table.

x	0	2	5	10	20	50	100
$100 - 9x$							
$100 - 8x$							
$100 - 7x$							
$100 - 6x$							
$100 - 5x$							
$100 - 4x$							
$100 - 3x$							

(b) Which sequence in the above table decreases fastest, and which sequence decreases slowest?

....

....

2. (a) Complete the table.

x	1	2	3	4	5	6	7
$2x + 3$							
$3x - 3$							
$3x - 2$							
$3x - 1$							

(b) For which value of x is $2x + 3$ equal to $3x - 1$? ....

(c) For which values of x is $2x + 3$ smaller than $3x - 1$?

....

....

(d) Do you think $2x + 3$ is smaller than $3x - 1$ for all values of x greater than 4? You may try a few numbers to help you think about this.

....

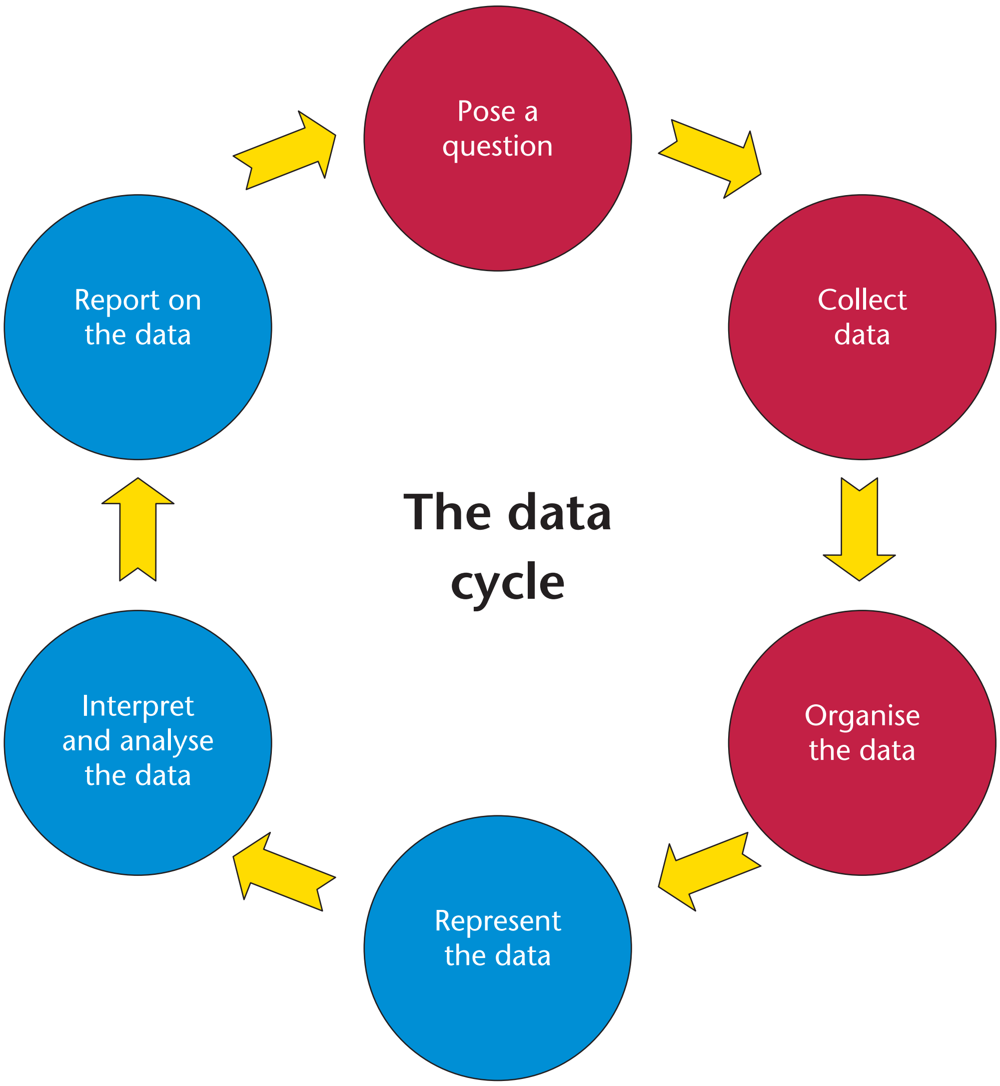
(e) Which sequence increases fastest, the sequence generated by $2x + 3$ or the sequence generated by $3x - 3$?

....

Chapter 13
Collect, organise and summarise data

Data handling is the part of Mathematics that deals with numbers and facts that we collect about the world around us. Data can be many different things, for example people's opinions on politics or the success rates of treating people with a certain kind of medicine. We use data to help us make decisions and solve problems about the world around us.

In this chapter you will focus on: collecting data using questionnaires; organising data, which includes using stem-and-leaf displays and grouping data into intervals; and then summarising data by determining the mode, median, mean and range of sets of numerical data.


Pose a question
Collect data
Organise the data
Represent the data
Interpret and analyse the data
Report on the data
The data cycle

13 Collect, organise and summarise data

13.1 Collecting data

Think of something that you really want to know about your own community or about children your age in other schools. For example, "How many Grade 7 learners in South Africa have access to a computer?" What would *you* find interesting to know about?

When you start the cycle of data handling, you start with at least one question. But there can of course be many more questions.

Once you have a research idea in mind, you can start planning how you will collect the data. When you collect data, you need to consider:

- what question you are asking
- where you will find the data to answer the question (for example from people such as your peers, family or the wider community; or from published sources such as newspapers, books or magazines)
- how you will collect the data (for example by using questionnaires or conducting interviews)
- who you will collect the data from (the entire population or a sample).

POPULATIONS AND SAMPLES: FROM WHOM TO COLLECT DATA

In data handling, **population** refers to the whole group you are asking the question about.
Sample refers to a small number of the group that you think will represent the whole group.

Here is an example: Thandeka wants to know about the home languages of all Grade 7s across the whole of South Africa. All Grade 7s in all of South Africa would be the population of that data. But it is not possible to reach every single Grade 7 learner in South Africa, so Thandeka could choose a sample of Grade 7 learners. For example, she could choose to collect data from her own Grade 7 class and from two other Grade 7 classes from two other schools.

But if Thandeka chose her own Grade 7 class and only two other classes from other schools, her sample would not really give information about learners across the whole of South Africa, because the learners in all three of the schools could be from the same language group.

So, how can you try to make sure that a sample gives information about the whole population? In other words, how can you make sure that your sample is **representative** of the population?

1. *Choose a big enough sample*. Generally, the bigger the sample is the more likely it is to represent the characteristics of the population.
2. *Ensure that you do not take a sample from only one of the groups within the population.* For example, if you want to find out if people like watching soccer, you cannot survey people at a Chiefs versus Pirates match. The majority of these people will almost certainly be there because they love watching soccer!

Example

Ganief wishes to find out if learners at his school like the style and colour of their school uniform and surveys 10 learners in Grade 7. There are 2 000 learners at the school.

Give two reasons to explain why the sample chosen is not likely to be representative of the population.

Answer

1. The sample is too small.
2. He is only getting the views of Grade 7s, not of the learners in any of the other grades (who might have very different views).

THINKING ABOUT POPULATIONS AND SAMPLES

1. Here are some research questions. Use a **P** to show which statement describes the population and an **S** to show which statement describes a sample of the population.

   (a) What percentage of plants in the vegetable patch is affected by disease?

   ☐ All the plants in the vegetable patch

   ☐ Every fourth or fifth plant in the vegetable patch

   (b) How often do teenagers recycle plastic?

   ☐ Every teenager in South Africa

   ☐ About 40 teenagers in the community

   (c) How many hours of sleep do 10-year-olds in my community get per night?

   ☐ All 10-year-olds in the community

   ☐ About ten 10-year-olds in the community

2. You want to know the most popular colour of the learners in your school.
   (a) Write down the population of your data collection.

   ....

   (b) Write down what sample you would use.

   ....

3. Census@School took place in 2001 and 2009. These were surveys that Statistics South Africa did to show learners how information about people is collected and analysed. The Census@School wanted to know personal, community and household information about learners from Grades 3 to 12. This is how they chose their sample:

   - *A sample of 2 500 schools was selected from the Department of Basic Education's database of approximately 26 000 registered schools.*
   - *The schools were divided into groups depending on their province, school type (primary: Grades 3 to 7 only; intermediate: Grades 5 to 9 only; secondary: Grades 8 to 12 only; combined: Grades 3 to 12), and education district.*
   - *A sample of schools was selected from each of these groups.*
   - *Approximately 790 000 learners participated in the Census@School 2009.*

   This information was included in their final report.

   (a) What percentage was the sample of all the schools in the country?

   ....

   (b) Why do you think they separated the schools into groups first?

   ....

   (c) Do you think the information that they obtained from this survey would be interesting to you? Explain.

   ....

   ....

4. Unathi goes to River View Girls' Primary School. She wants to find out whether 13-year-olds in her town prefer rugby or netball. She surveys 10 learners from each of the three Grade 7 classes at her school. Is the sample chosen likely to be representative of the population (13-year-olds in her town)? Explain your answer.

   ....

   ....

   ....

   ....

CONSTRUCTING QUESTIONNAIRES: HOW TO COLLECT DATA

A **questionnaire** is a sheet with questions used to collect data from people. Each **respondent** in the sample completes a questionnaire. The questions on the sheet can be structured differently, for example:

- The questions may require "yes" or "no" answers.
- A selection of answers (multiple-choice answers) may be provided for respondents to choose from.
- The respondents may enter their own views or information on the questionnaire.

A **respondent** is a person who fills in a questionnaire or from whom you collect data.

The type of responses you need (for example a simple "yes" or "no" or more detailed information) depends on the data you intend to collect.

Look at the examples below. Notice how each question is worded to be as clear as possible and to allow the data to be collected easily. (The questions in examples 4 and 5 were used by Census@School in their 2009 questionnaire.)

Example 1

Do you help with chores at home?

☐ Yes ☐ No

Example 2

Which of these chores do you help with?

☐ cleaning dishes ☐ washing clothes
☐ sweeping/vacuuming ☐ making beds

Example 3

How old are you?

☐ 5–8 years ☐ 9–11 years
☐ 12–15 years ☐ 16–19 years

Note in example 3 how all the ages from 5 to 19 are covered, but without any overlaps.

Example 4

14. Tick the box if you have:

1 ☐ Running water inside home
2 ☐ Electricity inside home
3 ☐ A radio at home
4 ☐ A TV at home
5 ☐ A telephone at home
6 ☐ A cell phone
7 ☐ Access to a computer
8 ☐ Access to the internet
9 ☐ Access to a library

Example 5

6. How tall are you without your shoes on? Answer to the nearest cm.

☐☐☐ centimetres

7. What is the length of your right foot, without a shoe? Answer to the nearest cm.

☐☐☐ centimetres

8. What is your arm span? (Open arms wide, measure the distance across your back from the tip of your right hand middle finger to the tip of your left hand middle finger.) Answer to the nearest cm.

☐☐☐ centimetres

MAKING QUESTIONNAIRES

1. (a) Refue wants to find out how much pocket money learners in her class receive each month. She draws up the following multiple-choice question:

How much pocket money do you get?

☐ 0–10 ☐ 10–20 ☐ 20–30 ☐ 30–40

Explain why this question is not clear. Give at least three reasons.

....

....

....

....

....

(b) Draw up the multiple-choice question so that it will allow Refue to collect the data that she needs.

2. You want to find out which sports learners at your school play.
   (a) Describe the population of your data.

   ....

   (b) Describe the sample you will use.

   ....

3. Make a question with yes/no or multiple-choice responses to help you collect the data you need:

4. Collect your data from your population or the sample you chose. Keep your data for the next chapter.

13.2 Organising data

To organise data that we have collected, we can use tally marks and tables, dot plots, and stem-and-leaf displays. We can also group the data when there are many data values. The ways that we organise the data depends on the type of data we collected.

DIFFERENT TYPES OF DATA

Look at the five examples of questions for questionnaires on pages 172 and 173.

1. Which of the examples will give you data that looks like this?

Yes	1 235 learners
No	1 265 learners

....

2. Which of the examples might give you data that looks like this?
132 cm; 141 cm; 160 cm; 132 cm; 154 cm; 145 cm; 147 cm; 129 cm; 121 cm; 143 cm; 135 cm; 154 cm; 156 cm; 133 cm; 156 cm; 123 cm; 137 cm etc.

....

3. What could the data for example 4 look like? Fill in this table to give a possible example for 30 learners. Use numbers that you have made up.

	Number of learners

4. Which of the examples might give you a data set that looks like this?

5–8 years	15
9–11 years	45
12–15 years	32
16–19 years	28

....

The type of data in questions 1 and 3 is called **categorical data**. This is often described by words. The categories don't have to be given in order.

The type of data in questions 2 and 4 is called **numerical data**. Numerical data can be whole numbers only, or it can include fractions.

For both of these kinds of data, your results give you a list of responses. You will soon learn how to organise these responses.

5. Classify the following data sets as categorical or numerical.
   (a) the number of pages in books ....
   (b) the length of learners' arm spans ....
   (c) learners' favourite soccer teams ....
   (d) the time it takes 13-year-olds to run 1,5 km ....
   (e) the cost of different types of cell phone ....
   (f) colours of new cars manufactured ....

ORGANISING CATEGORICAL DATA

Thandeka asked the following question: "Which of South Africa's official languages are the home languages of the learners in my class?"

Thandeka drew up a table with each learner's name. She then asked each learner what his or her home language was, and wrote it down as follows:

Name	Language	Name	Language	Name	Language
Nonkhanyiso	isiXhosa	Marike	Afrikaans	Herbert	Sepedi
Anna	Afrikaans	Jennifer	Sepedi	Thabo	isiXhosa
Mpho	Ndebele	Nomonde	isiXhosa	Nomi	isiXhosa
Nontobeko	isiZulu	Thandeka	Sepedi	Manare	Sepedi
Jonathan	English	Siza	isiZulu	Unathi	Sesotho
Sibongile	isiZulu	Prince	Sesotho	Gabriel	Ndebele
Dumisani	isiZulu	Duma	isiZulu	Marlene	Afrikaans
Matshediso	Sesotho	Thandile	Sepedi	Simon	Sesotho
Chokocha	Sepedi	Nicholas	Sesotho	Miriam	Setswana
Khanyisile	isiXhosa	Jabulani	isiZulu	Sibusiso	isiZulu
Ramphamba	Tshivenda	Nomhle	isiXhosa	Mishack	isiZulu
Portia	isiZulu	Frederik	Afrikaans	Peter	Setswana
Erik	Afrikaans	Lola	Afrikaans	Maya	Afrikaans
Jan	Afrikaans	Zinzi	isiXhosa	Thobile	Sesotho
Palesa	isiZulu	Jacob	Setswana		

We don't need the learners' names in the data. This data could be written as a list of the languages, like this:

isiXhosa, Afrikaans, Sepedi, Afrikaans, Sepedi, isiXhosa, Ndebele, isiXhosa, isiXhosa, isiZulu, Sepedi, Sepedi, English, isiZulu, Sesotho, isiZulu, Sesotho, Ndebele, isiZulu, isiZulu, Afrikaans, Sesotho, Sepedi, Sesotho, Sepedi, Sesotho, Setswana, isiXhosa, isiZulu, isiZulu, Tshivenda, isiXhosa, isiZulu, isiZulu, Afrikaans, Setswana, Afrikaans, Afrikaans, Afrikaans, Afrikaans, isiXhosa, Sesotho, isiZulu, Setswana

Now work with this data set to see what story it is telling you. What do you notice about the data?

1. What do you need to find out from this list of languages?

2. Does it matter what order you write the languages in? Why or why not?

3. (a) Use Thandeka's table. In the space below, draw a dot above each language to show every learner who speaks that language. The languages are in alphabetical order. Try to space out the dots evenly. The dots for Afrikaans have been drawn for you. A graph like this is called a **dot plot**.

(b) Which languages have the same numbers of learners?

(c) List the languages in order from the language spoken by the most learners to the language spoken by the fewest learners.

You can also record results in a **tally table**. To do this, you draw a single line (|) for each item you count. This line is called a **tally mark**.

You group tally marks in groups of five. The fifth tally mark is always drawn horizontally to show that the group of five is complete. Then you start a new group. This makes it easy to quickly count how many tally marks there are in a particular category.

Examples of tally marks:
A count of three = |||
A count of four = ||||
A count of five = ~~||||~~
A count of seven = ~~||||~~ ||

4. (a) Complete the table.

Home language of learners in the Grade 7 class

Language	Number of speakers of each home language	Total
Afrikaans	~~||||~~ |||	8
English			1
isiXhosa		
isiZulu		
Ndebele		
Sepedi		
Sesotho		
Setswana		
Siswati		
Tshivenda		
Xitsonga		
Total (whole class)		

(b) How many learners altogether were asked about their home language? ....

(c) Which home language occurs most often in this class? ....

(d) Which languages are not spoken as a home language by any of the learners in this class?

....

(e) Write a short paragraph to describe the home languages in Thandeka's class.

....

....

....

....

Maths2_Gr7_LB_Book.indb 178 2014/09/04 11:35:22 AM

Dot plots and tally tables are used for **numerical data** too. You can write data values on prepared tally tables or dot plots as you record them. This sorts the data at the same time as it is recorded.

INTRODUCING STEM-AND-LEAF DISPLAYS

A **stem-and-leaf display** (also called a stem-and-leaf plot) is a way of listing numerical data using two columns divided by a vertical line. Each number is split across the columns.

For example, if the numbers in a set of data consist of digits for tens and units (such as 23, 25, 34), the column on the right (the leaf column) shows the units digits of the numbers, and the column on the left (the stem column) shows the tens digits of the numbers.

Numerical data is data that consists of numbers.

Example 1

Show the following data set as a stem-and-leaf display:

13, 56, 20, 35, 47, 53, 12, 51, 53, 49, 34, 53

First, we order the values in the data set from smallest to biggest:

12, 13, 20, 34, 35, 47, 49, 51, 53, 53, 53, 56

In this example, the tens digits range from 1 to 5, so we list these in the stem column. Then we fill in the units digits in the leaf column.

The stem-and-leaf display of the above data set looks like this:

Key: 1 | 2 means 12

Stems	Leaves
1	2, 3
2	0
3	4, 5
4	7, 9
5	1, 3, 3, 3, 6

Values with the same stem are written in the same row. Numbers with the same tens value are separated by a space or a comma. The first row shows the numbers 12 and 13, the second row shows 20, and the last row shows 51, 53, 53, 53 and 56.

In this example, the stem column shows the tens digit of each value.

Here the leaf column shows the units digit of each value.

Example 2

This stem-and-leaf display shows the units digits as the leaves, and both the hundreds and tens digits as the stems:

10	2, 5
11	0, 6
12	1, 4, 4
13	
14	7, 9
15	
16	1, 3, 8

Key: 10 | 2 means 102

The values shown are: 102, 105, 110, 116, 121, 124, 124, 147, 149, 161, 163, 168.

Note that if there is a 0 in the leaf column it means the unit digit is a 0, as in 110 above. When there is nothing written in the leaf column next to a stem, it means that there aren't any numbers with that particular stem. In the case of stem 13 above, for example, it means there are no values between 129 and 140.

When you draw stem-and-leaf displays, it is important that the numbers line up vertically so that you can compare the leaves. Draw lines to help you. (Or use grid paper, if you have some.)

DOT PLOTS AND STEM-AND-LEAF DISPLAYS

1. Look at the following stem-and-leaf display and answer the questions below.

13	1, 9
14	0
15	
16	2, 3, 5, 5, 5
17	6, 8, 8
18	
19	4, 6, 7

Key: 13 | 1 means 131

(a) Write down the values in the data set shown by the stem-and-leaf display.

....

(b) Do most of the values fall in the 160s or 170s? ....

(c) Which value occurs the most times? ....

(d) Add the following values to the stem-and-leaf display: 143, 167 and 199.

(e) There are no values in the 150s. Can we add the following to the stem-and-leaf display to show that there are no values in the 150s? Explain your answer.

15 | 0

....

....

2. (a) Arrange the values in the following data set in order from smallest to largest:
378, 360, 390, 378, 378, 400, 379, 382, 354, 394, 399, 395, 378, 361, 375

....

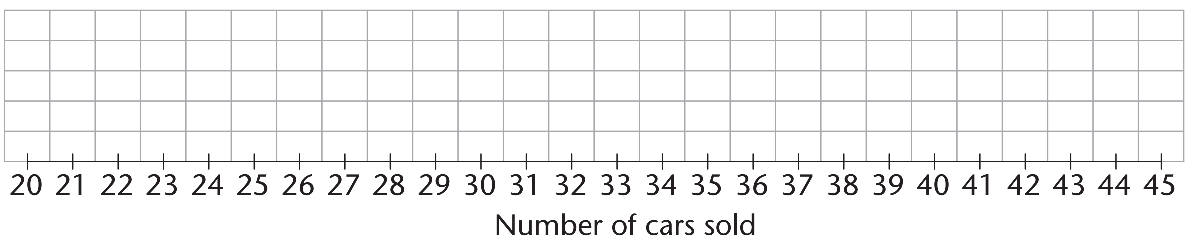
(b) Organise the data set as a stem-and-leaf display.

Key: ....

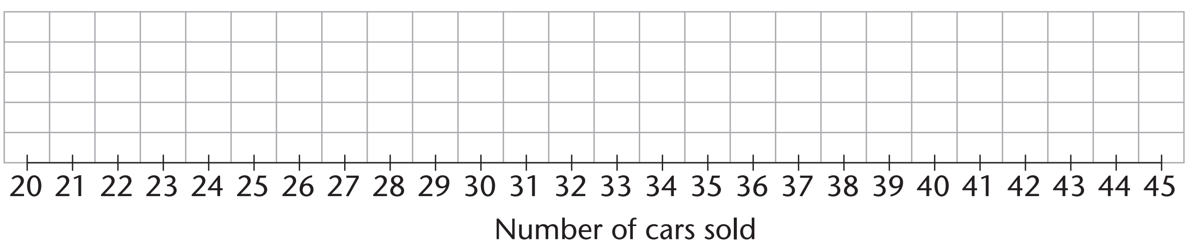
(c) Which value occurs most often? ....

3. (a) The data sets below show the sales of two new makes of cars (Jupiter and Mercury) over 24 months. Draw a dot plot for each set on the number lines.
Mercury: 23, 27, 30, 27, 32, 31, 32, 32, 35, 33, 28, 39, 32, 29, 35, 36, 33, 25, 35, 37, 26, 28, 36, 30
Jupiter: 31, 44, 30, 36, 37, 34, 43, 38, 37, 35, 36, 34, 31, 32, 40, 36, 31, 44, 26, 30, 37, 43, 42, 33

Mercury



Jupiter



(b) If you look at the dots for Mercury and the dots for Jupiter, what can you see about the sales of the two cars? What does this mean?

....

....

....

SOMETHING TO THINK ABOUT

What kind of graph does the stem-and-leaf display look like if you turn it by 90°?

1	2, 5
2	0, 6
3	1, 4, 4, 5, 5, 9
4	
5	2, 7, 8, 9
6	1, 3, 6, 7, 7
7	1, 3, 8

→

		9				
		5			7	
		5		9	7	
		4		8	6	8
5	6	4		7	3	3
2	0	1		2	1	1
1	2	3	4	5	6	7

GROUPING DATA INTO INTERVALS

When a data set contains many data items, we sometimes group the data items to help us organise the data. For example, the following data set shows the number of milk bottles collected by 24 learners for recycling:

9, 10, 13, 23, 24, 26, 26, 27, 30, 31, 34, 40, 42, 49, 50, 53, 61, 64, 67, 67, 68, 69, 91, 94

We can group the data into categories called **class intervals**, such as 0–9, 10–19, 20–29, and so on. We can then count how many times a value occurs in each interval. The number of times a value occurs in an interval is called its **frequency**.

This table shows the grouped data and the frequency of the values in each interval.

Interval	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–99
Frequency	1	2	5	3	3	2	6	0	0	2

The table shows that 1 learner collected 0–9 bottles, 2 learners collected 10–19 bottles, 5 learners collected 20–29 bottles, and so on. We can clearly see that most learners (6) collected 60–69 bottles.

WORKING WITH GROUPED DATA

1. Anita collected data from a sample of Grade 7 learners about how far they live from the nearest grocery store. Below are the results. The values are in kilometres, correct to one decimal figure.

0,1	0,1	0,2	0,2	0,2	0,2	0,3	0,3	0,3	0,4	0,4	0,5	0,5	0,5	0,6
0,6	0,7	0,7	0,7	0,8	0,8	0,8	0,9	0,9	0,9	1	1	1	1,5	1,5
2	2	2	2	2,5	2,5	3	3	3	3,5	3,5	4	4	4,	4,5
5	5	6	6	7	7	8	8	9	10	10	15	20	23	30

(a) Complete the table alongside to indicate how many of the values appear in each of the given intervals.

(b) How far do most of the learners live from the nearest grocery store?

....

Interval	Frequency
less than 1,0 km	
1,0–5,9 km	
6,0–9,9 km	
10 km or further	

2. Here are the heights of 50 Grade 7 boys at a school (in centimetres):

165	148	150	160	165	150	156	155	164	162
160	158	138	158	140	146	160	148	152	139
165	148	152	139	165	148	160	163	178	138
142	179	156	160	160	171	140	160	164	135
159	143	167	138	163	164	155	160	167	165

(a) Draw a stem-and-leaf display to show this set of data.

Key:

(b) Write a short paragraph to describe the data set.

....

....

....

(c) Complete the frequency table below for the grouped data from your stem-and-leaf display in question (a).

Class interval (cm)	Frequency
130–139	
140–149	
150–159	
160–169	
170–179	
Total	

13.3 Summarising data

When you have collected data, you often need to tell someone what you have found out. People want to know what your conclusions are, without looking at all of the data you have collected.

It is often useful to summarise a set of numerical data by using *one* value. For example, which value best summarises or describes the following data set?

0 1 1 5 8 8 9 9 10 10 10 11 11

Statisticians use any of three values that show the most central values in the set, or the value around which the other values tend to cluster. These values are called the **measures of central tendency** or **summary statistics**.

Statisticians are mathematicians who specialise in collecting, organising and analysing data.

- The **mode** is the value that occurs the most frequently in the data set. In the example above, the mode is 10 because it occurs the most times (three times).
- The **median** is the value exactly in the middle of the data set when the data values are arranged in order from smallest to largest. For the data set above, the median is 9 because there are six values to the right of the first 9 and six values to the left of it.
- The **mean (average)** is the total (sum) of the values divided by the number of values in the data set. So:

$$\text{Mean} = \frac{\text{Total of values}}{\text{Number of values}} = \frac{93}{13} = 7{,}15$$

A data set can have more than one mode.

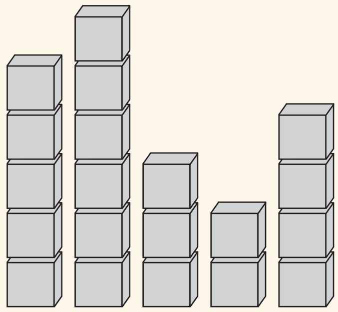
If the data set consists of an even number of items, the median = sum of the two middle values divided by 2.

In the data set above, either 10 (mode), 9 (median) or 7,15 (mean) could be used to represent the entire data set.

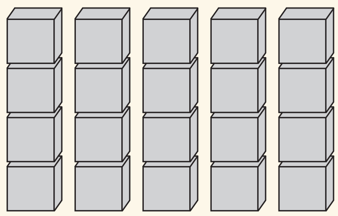
UNDERSTANDING THE MEAN

This activity will help you to understand how the mean represents the whole set of data.

Make piles of blocks of different heights:



Then move blocks from the higher piles to the lower ones to make all the piles equal:



You have just found the mean: Each pile now has 4 blocks in it. But how do you do this if you only have the numbers 5, 6, 3, 2 and 4 to work with? You add them up and then divide the answer by the total number of values (numbers):

$5 + 6 + 3 + 2 + 4 = 20$ $\quad\quad$ $20 \div 5 = 4$

What this means is that you are finding a single number that you can use in place of all the different numbers and still get the same total.

It is also useful to know how big the spread of the data is.

The **range** of a data set is the difference between the highest value and the lowest value. For example, for the data set on the previous page, the range is:

$11 - 0 = 11$

The bigger the range, the more the data is spread out. The smaller the range, the more the data is clustered around similar values.

DETERMINING THE MODE, MEDIAN, MEAN AND RANGE

1. The following data set shows the shoe sizes of a sample of learners at a school:
   1, 1, 1, 2, 2, 2, 3, 3, 4, 4, 4, 5, 5, 5, 5, 5, 5, 6, 6
   (a) What is the mode of the data set? ....
   (b) What is the median of the data set? ....
   (c) What is the mean? (Round off to the nearest whole number.)
   ....
   ....
   (d) What is the range of the data set?
   ....

2. The following data set shows the number of siblings (that is, brothers and sisters) that the learners in a sample of Grade 7 learners have:
   0, 0, 0, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 4, 4, 5
   (a) How many learners are in the sample? ....
   (b) What is the mode of the data set? ....
   (c) What is the median of the data set? ....
   (d) What is the mean? (Round off to the nearest whole number.)
   ....
   ....
   ....
   (e) What is the range of the data set? ....

3. The following data set shows the number of hours worked in a week by a sample of parents at School A:
   15, 16, 20, 25, 25, 30, 40, 40, 40, 40, 40, 42, 45, 45, 48, 48
   (a) How many parents are in the sample? ....
   (b) What is the mode of the data set? ....
   (c) What is the median of the data set?
   ....
   (d) What is the mean? (Round off to one decimal place.)
   ....
   (e) What is the range of the data set? ....

Remember, if the number of items in a data set is even, the median = the sum of the two middle numbers divided by 2.

4. The following data set shows the number of hours worked in a week by a sample of parents at School B:
25, 30, 35, 35, 35, 40, 40, 40, 40, 40, 42, 45, 45, 45, 48, 50
   (a) How many parents are in the sample? ....
   (b) What is the mode of the data set? ....
   (c) What is the median of the data set?

....

....

....

   (d) What is the mean? (Round off to one decimal place.)

....

....

....

   (e) What is the range of the data set?

....

5. The following is a list of test scores of learners in a Grade 7 class:
40, 42, 44, 13, 10, 23, 68, 31, 69, 91, 30, 49, 50, 53, 67, 94, 61, 64, 67, 34
   (a) Arrange the scores from the lowest to the highest.

....

   (b) How many learners are in the population? ....
   (c) What is the mode of the data set? ....
   (d) What is the median of the data set?

....

....

....

   (e) What is the mean?

....

....

....

   (f) What is the range of the data set?

....

6. A hockey player recorded the number of goals she scored in her last 30 matches:

1 1 3 2 0 0 4 2 2 4 3 1 0 1 0

2 1 5 1 3 7 2 2 2 4 3 1 1 0 3

(a) Draw a dot plot on the number line below to organise these data values.

Now use the dot plot to answer these questions.

(b) Which of the values are quite different to the other values?

....

(c) Which number of goals has she scored the highest number of times?

....

(d) Which numbers of goals did she score in the two groups with five matches each?

....

(e) Use the dot plot to find the mode of the data.

....

(f) Use the dot plot to find the median.

....

....

(g) What is the mean of the goals?

....

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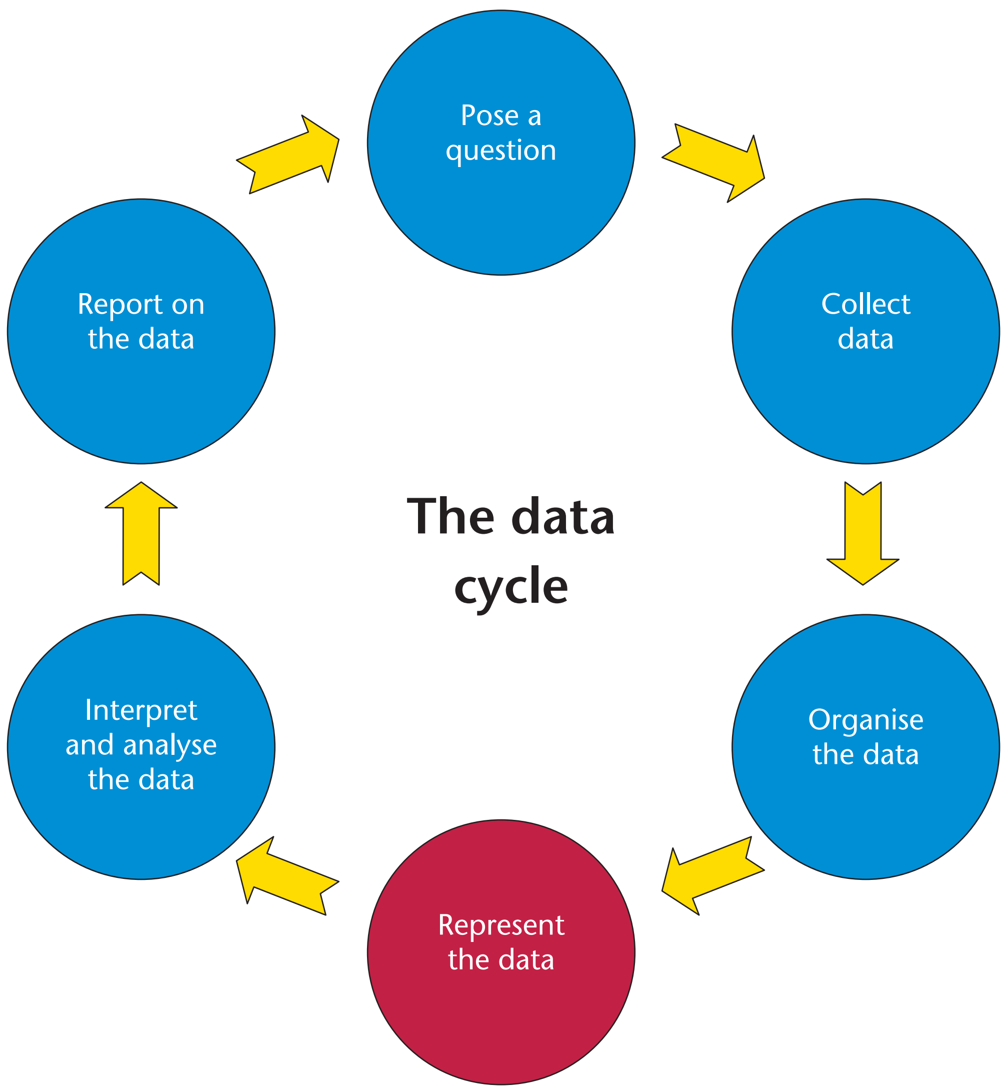
Chapter 14
Represent data

When we have collected and organised our data, we often represent it as a graph. This helps us to see the data and patterns in the data more easily. In this chapter, you will revise bar graphs, double bar graphs and pie charts, which you have learnt about in previous grades. You will learn about a new type of graph called a histogram, and how this differs from a bar graph. You will also learn how to draw your own pie charts by estimating fractions of a whole circle.


Pose a question
Collect data
Organise the data
Represent the data
Interpret and analyse the data
Report on the data
The data cycle

14 Represent data

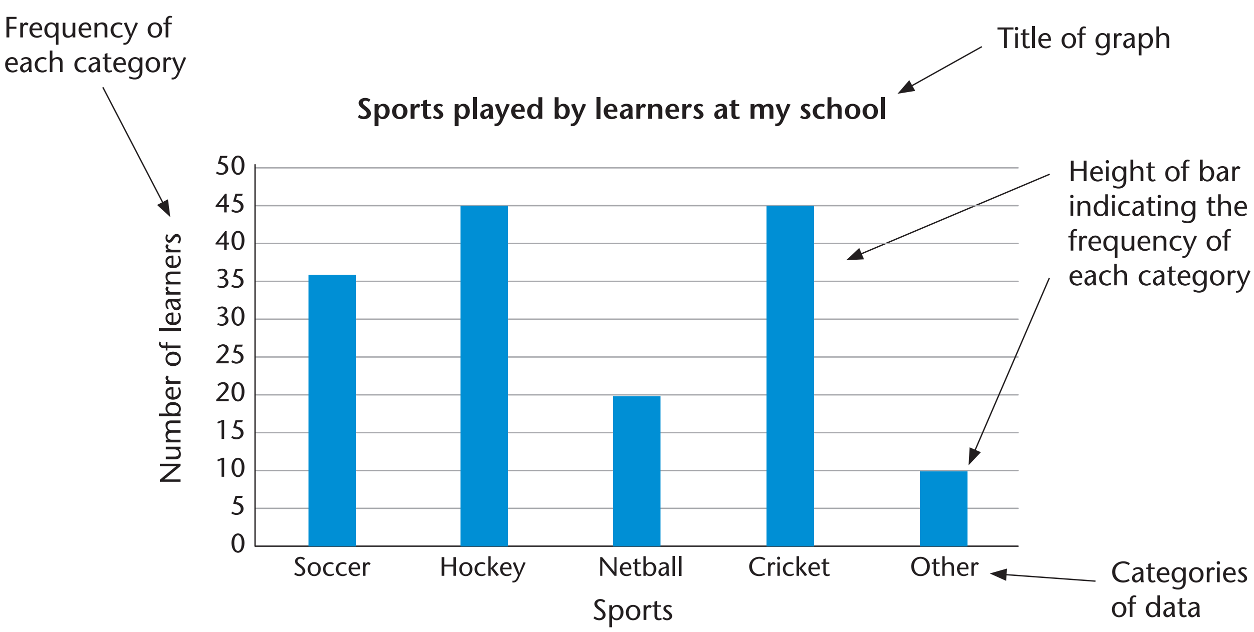
Now that we have collected and organised a set of data, we want to show the results in a useful way.

Remember when you drew dot plots in the previous chapter, you could see which categories or measurements occurred many times and which occurred only a few times. There are a few different graphs that show the important things about the data in such a way that you can see them easily. You need to be able to draw these graphs.

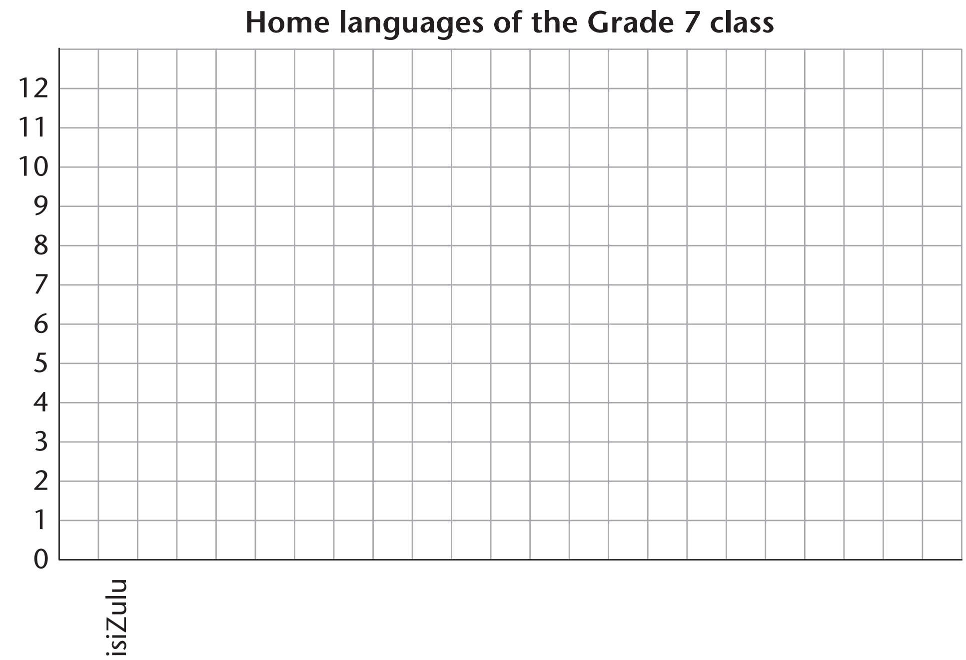
14.1 Bar graphs and double bar graphs

DRAWING A BAR GRAPH

A **bar graph** shows categories (or classes) of data along the horizontal axis, and the frequency of each category along the vertical axis. (Sometimes the axes are swopped around.) Here is an example of a bar graph.

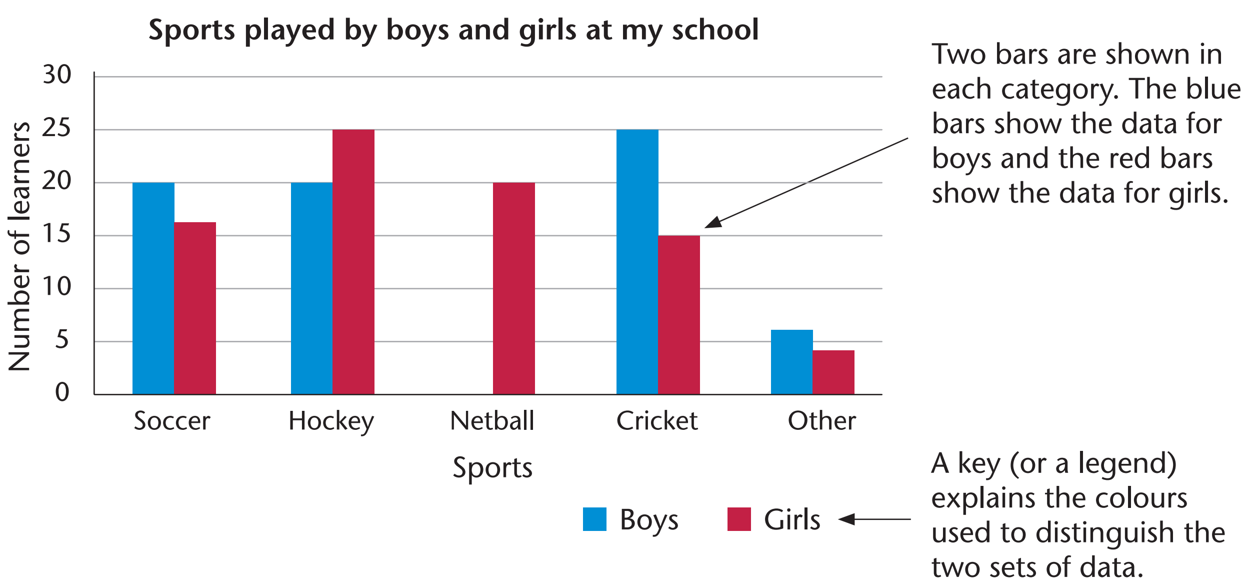


Go back to section 13.2 of chapter 13, where you drew a dot plot and made a tally table of Thandeka's data about languages spoken in her class. Use the data to draw a bar graph on the set of axes on the next page. Draw the bars to the correct height by looking at the numbers on the vertical axis.



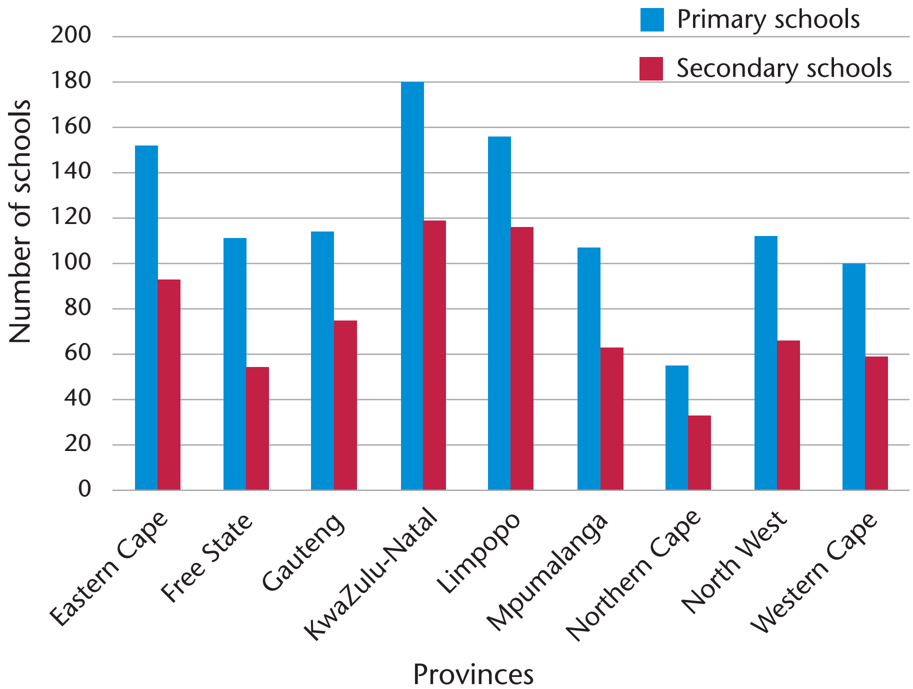
USING DOUBLE BAR GRAPHS

A **double bar graph** shows two sets of data for each category (or class). For example, the double bar graph below shows data collected from girls for each category, and data collected from boys for each category.



1. Look at the data below and answer the questions that follow.

Number of schools, by province, participating in a school survey



(a) Did more primary schools or more secondary schools participate in the survey?

....

(b) Which province had fewer than 50 secondary schools participating in the survey?

....

(c) Which provinces had more than 150 of its primary schools participating in the survey?

....

2. Draw a double bar graph to show the following data. Use the grid on the next page.

Facilities available at schools in Province A and Province B

Facility	Percentage of schools in Province A	Percentage of schools in Province B
Electricity	73	50
Running water	68	45
Computers	60	20
Internet	30	10

14.2 Histograms

A SITUATION WHERE DATA HAS TO BE ORGANISED

1. Mr Makae wants to buy an orange farm. Three farms are available, each with an orchard of orange trees, and the three farms cost about the same. There are 40 orange trees on each farm. The total mass of oranges (in kg) harvested from each tree on each farm over the last 3 years is given below. Which farm should he buy?

Farm A:

426	628	467	413	862	585	652	600	734	611
741	605	536	643	833	438	613	704	623	719
719	701	501	768	642	444	751	579	695	726
616	619	441	703	902	947	785	952	725	721

Farm B:

822	736	773	674	884	463	644	433	688	487
884	530	448	410	982	638	492	638	725	621
743	661	744	530	560	745	455	943	760	734
888	457	621	969	507	500	542	831	576	801

Farm C:

438	530	743	947	450	777	859	748	473	724
750	852	428	464	725	554	758	997	467	743
722	438	779	690	785	543	752	898	474	483
460	772	544	756	491	576	482	744	701	803

2. How can the data about the orange trees on the three farms be organised so that the farmer has a clear picture of the difference between the orchards on the three farms? For now just write down how you think the data may be organised. You will organise the data later when you do the questions that follow.

....

....

3. Complete these tally and frequency tables for the data about the masses of oranges harvested on the three orange farms.

Masses of oranges harvested from different trees on Farm A

Mass of oranges harvested from each tree. These are called **class intervals**.	Number of trees that produced masses in the interval	Total
400 kg or more but less than 500 kg	𝍸 |	
500 kg or more but less than 600 kg	||||	
600 kg or more but less than 700 kg	𝍸 𝍸 ||	
700 kg or more but less than 800 kg	𝍸 𝍸 |||	

Masses of orange harvested from different trees on Farm B

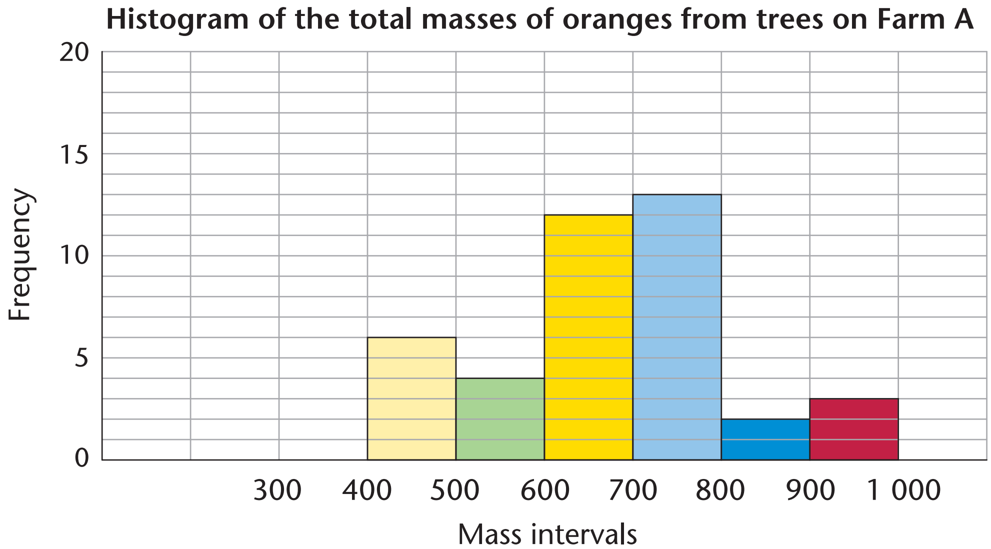
Class interval	Number of trees that produced masses in the interval	Total
400 kg or more but less than 500 kg	𝍸 |||	8
500 kg or more but less than 600 kg	𝍸 ||	7
600 kg or more but less than 700 kg	𝍸 |||	
700 kg or more but less than 800 kg	𝍸 |||	
800 kg or more but less than 900 kg		
900 kg or more but less than 1 000 kg		

Masses of oranges harvested from different trees on Farm C

Class interval	Number of trees that produced masses in the interval	Total
400 kg or more but less than 500 kg		
700 kg or more but less than 800 kg		

On the next page, you will learn how to draw graphs of the data for the three farms.

The data for Farm A is represented on this graph.



This type of graph is called a **histogram**.

(The columns in a histogram are normally not coloured differently, or even coloured at all. In this histogram the columns are coloured only because some questions are asked about them in question 4 below.)

The numbers 400 on the left and 500 on the right of the light yellow column indicate that masses of 400 kg or more but less than 500 kg are counted in that interval.

The height of each column represents the number of masses (the frequency) that fall in that interval.

4. (a) A total of 536 kg of oranges was harvested from one of the trees on Farm A over a period of the 3 years. In which column on the above histogram is this tree represented? Explain your answer.

....

....

....

(b) Which masses are represented in the red column?

....

....

(c) Which class interval is represented by the light blue column on the above histogram?

....

....

(d) How many masses are represented by the green column?

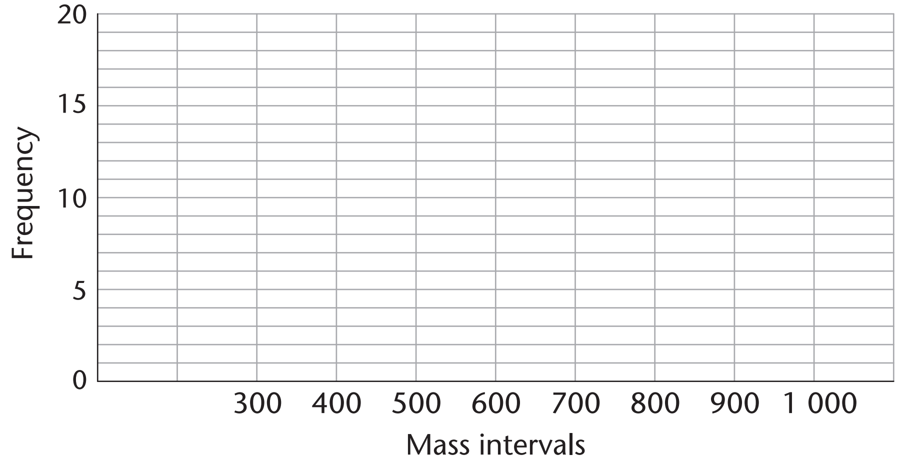
....

(e) Which column represents the highest frequency?

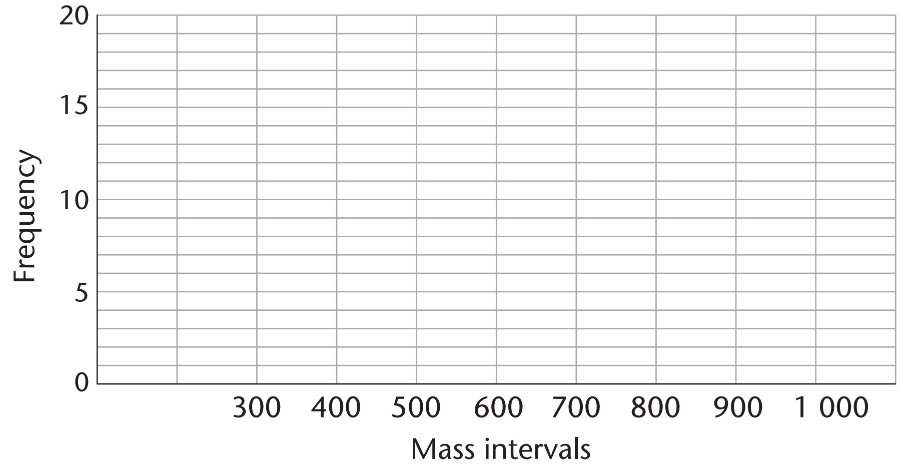
....

5. Complete the histograms below.

Histogram of the total masses of oranges from trees on Farm B



Histogram of the total masses of oranges from trees on Farm C



The different class intervals are **consecutive** and cannot have values that overlap. For example, we can group heights into class intervals of 10 cm, as shown below:

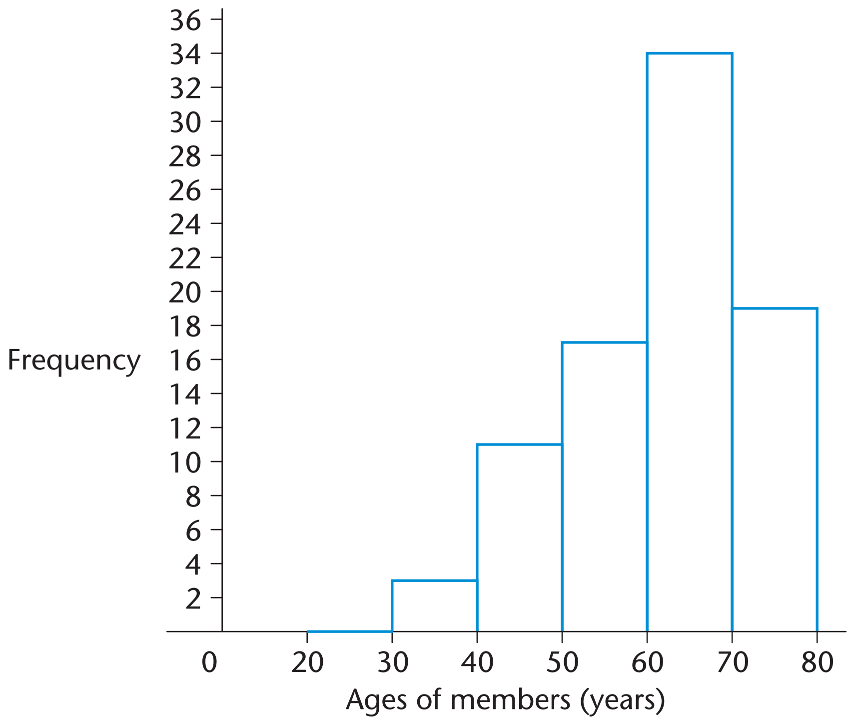
Height (m)	Heights that fall in the class interval	Frequency
1,20–1,30	1,20; 1,25; 1,29	3
1,30–1,40	1,30; 1,31; 1,35; 1,39	4
1,40–1,50	1,40; 1,46; 1,48; 1,48; 1,49	5
1,50–1,60	1,53; 1,53; 1,57; 1,58; 1,59; 1,59	6

We follow the convention that the top value (also called the **upper boundary**) of each class interval is not included in the interval.

So the height of 1,20 m falls into the 1,20–1,30 m interval, but the height 1,30 m falls into the 1,30–1,40 m interval.

INTERPRETING A HISTOGRAM

Study the histogram showing the numbers of members, in different age groups, of a sports club. Then answer the questions that follow.



1. Complete a frequency table for the information.

2. How many of the members are in their fifties? ....

3. How many members does the club have?

....

4. When you drew a bar graph, it did not matter what order the bars were in. Does the order of the columns on the histogram matter? Explain.

....

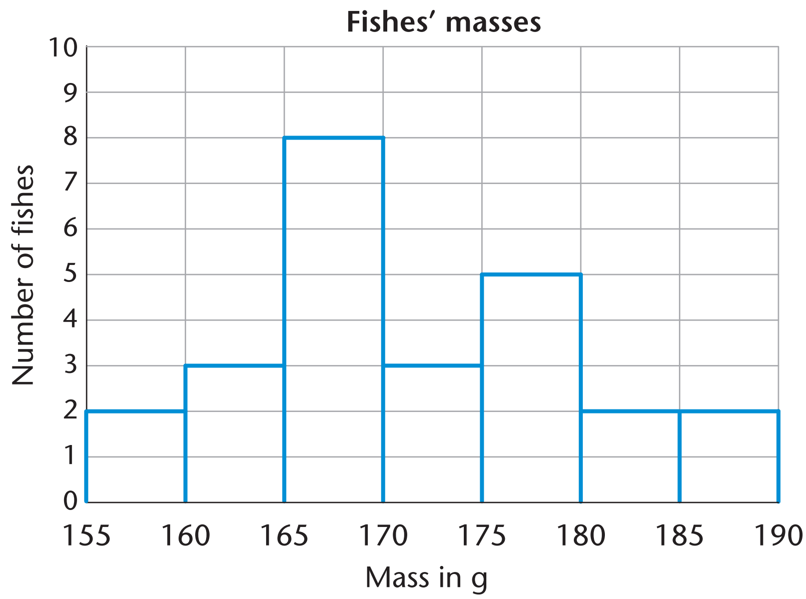
....

Notice that you cannot see the individual data values in a histogram – they have been "lost". For example, below you can see a stem-and-leaf display and a histogram of the same data set:

Fishes' masses in g

15	7, 8
16	2, 3, 3
16	5, 5, 6, 7, 8, 8, 9, 9
17	1, 2, 3
17	6, 7, 7, 8, 8
18	0, 3
18	6, 7

Key: 15|7 means 157



A histogram usually has many more data values than a stem-and-leaf display – too many to show in a stem-and-leaf display. It would, for example, be difficult to put the 84 values for the members of the sports club onto a stem-and-leaf display.

DRAWING MORE HISTOGRAMS

1. The table shows how long it takes learners from a Grade 7 class at Western Primary to travel to school each day. In question (d) you will represent the data in the table with a histogram.

Time (minutes)	Frequency
0–10	7
10–20	18
20–30	11
30–40	3

   (a) How many learners were asked about their travelling hours?

   ....

   (b) Look at the grid provided in question (d). What do you have to consider in order to help you decide on a scale division for the vertical axis?

   ....

   ....

   (c) What scale will you use on the horizontal axis? Explain your answer.

   ....

   ....

(d) Draw a histogram of the data.

2. The table shows how much money different vendors earn selling their goods every week.

Money (R)	Frequency
0–100	6
100–200	9
200–300	11
300–400	7
400–500	5

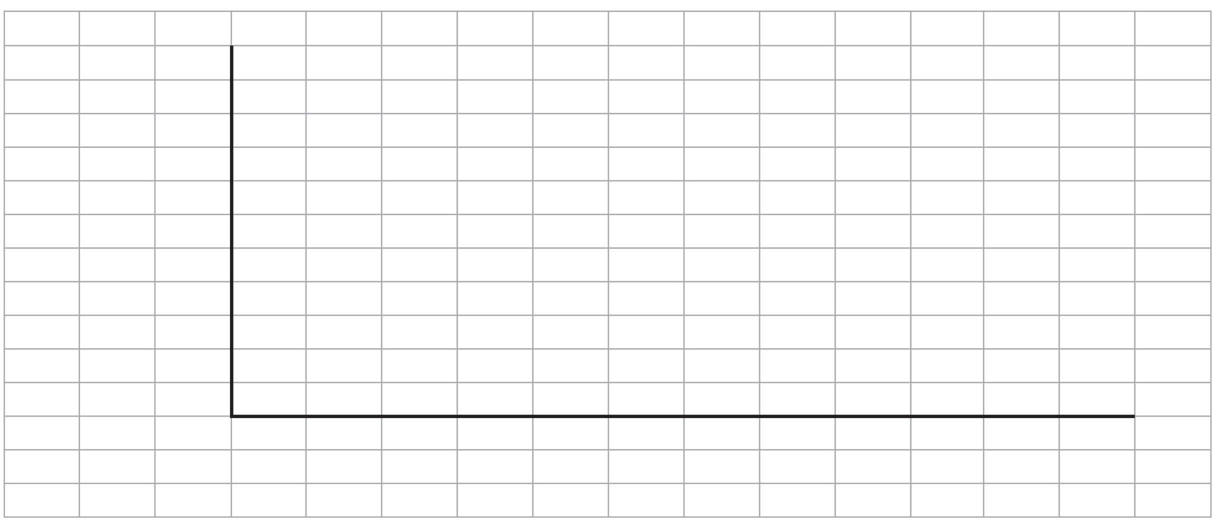
(a) How many vendors were asked about their earnings?

....

(b) Look at the grid below. Decide on a scale for the vertical axis of a histogram and indicate it on the axis.

(c) Decide on a scale for the horizontal axis and indicate it on the axis.

(d) Complete the histogram showing the data.



3. In a Natural Sciences class, learners planted beans and measured the heights of the bean plants after two months. Here is the data they collected (in cm):

34 65 72 42 37 29 78 43 79 91 43 45 28 42 79
34 92 87 40 43 43 78 82 47 85 43 32 86 76

(a) Complete this frequency table:

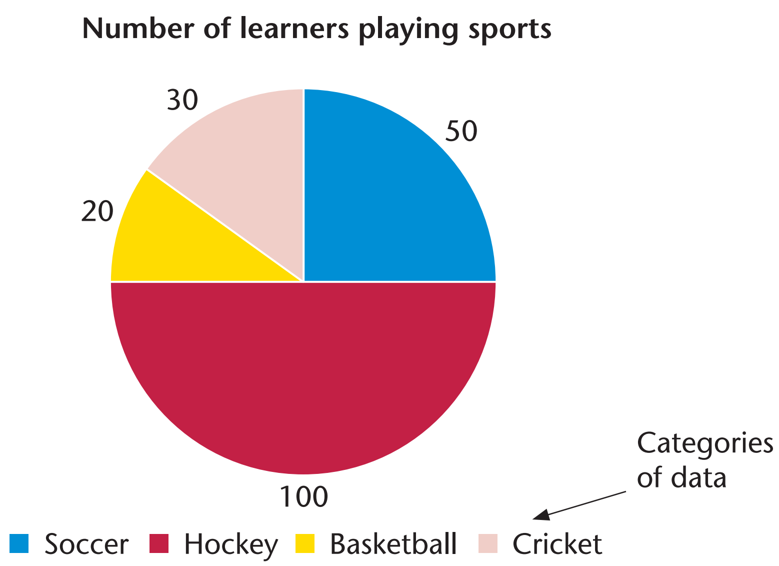
Height of bean plants (cm)	Tally	Frequency
20–30		
30–40		
40–50		
50–60		
60–70		
70–80		
80–90		
90–100		
Total		

(b) Draw a histogram of this data.

14.3 Pie charts

A **pie chart** consists of a circle divided into slices (**sectors**), where the slices show how the different categories of data make up the whole set of data. Bigger categories of data have bigger slices of the circle.

Look at the example of a pie chart below.



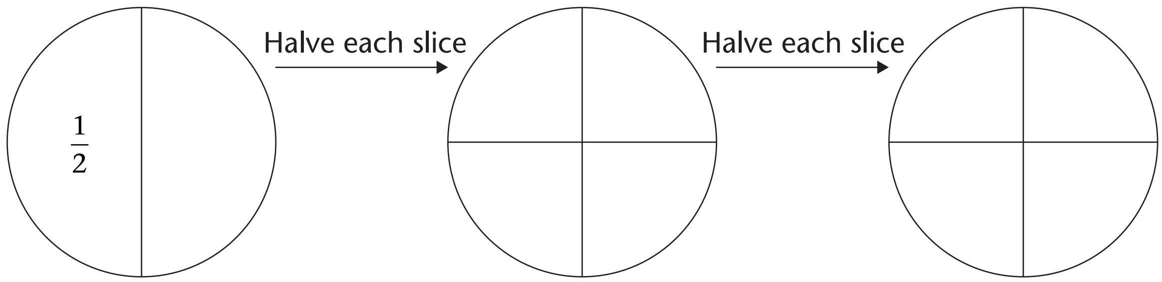
The pie chart shows the following:

- A total of 200 learners were asked about the sports they played: 20 + 30 + 50 + 100 = 200
- The key shows the four categories of data:
  - soccer
  - hockey
  - basketball
  - cricket.
- 100 of the 200 learners play hockey. This is the largest category, and gets the biggest slice (half of the whole).
- 20 of the 200 learners play basketball. This is the smallest category, and gets the smallest slice (one tenth of the whole).

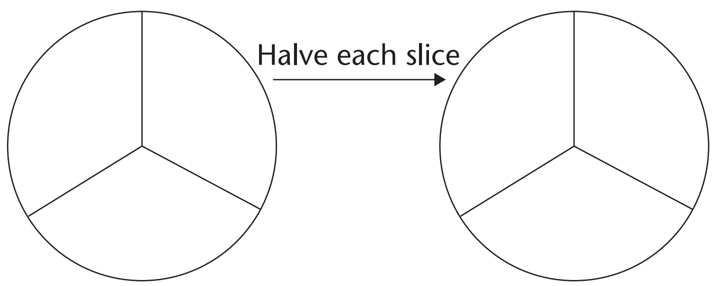
You will learn how to draw accurate pie charts in later grades. In this grade, you will estimate the portions of a pie chart that each category of data requires.

ESTIMATING SIZES OF SLICES IN A PIE CHART

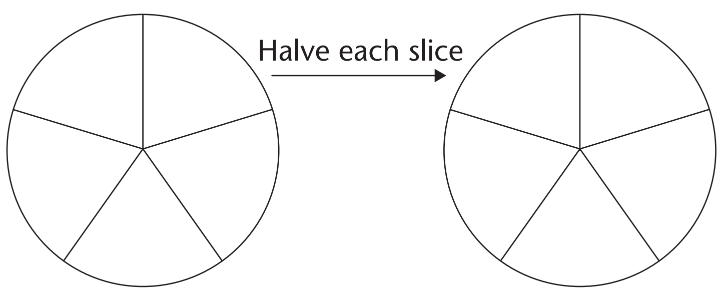
1. (a) Write down the fraction of a whole that each slice in the following diagrams shows.



$\frac{1}{2}$ = ... % ....



....



....

(b) Below each diagram in question 1(a), write down what percentage each fraction is equal to.

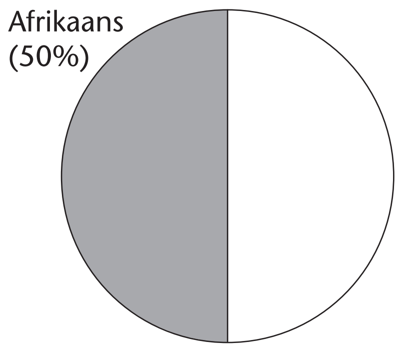
You can use the diagrams above to estimate the sizes of slices when drawing your own pie charts.

2. Use the data in each of the following tables to complete the pie charts. You must:
   - label the major sector
   - divide the other sector into the parts that represent the other languages
   - label each sector.

(a)

Province: Western Cape

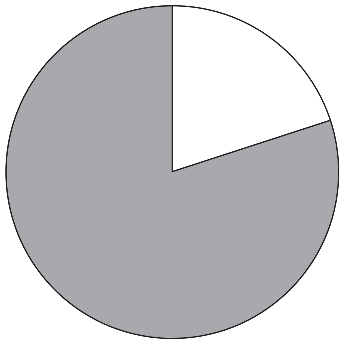
Major languages	Frequency (in %)
Afrikaans	50%
English	20%
isiXhosa	25%
Other	5%



(b)

Province: KwaZulu-Natal

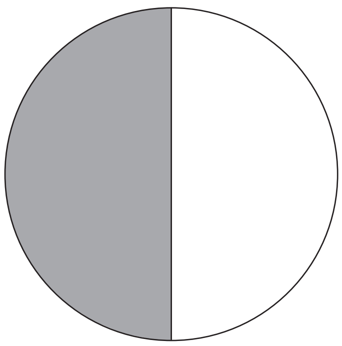
Major languages	Frequency (in %)
English	15%
isiZulu	80%
Other	5%



(c)

Province: Limpopo

Major languages	Frequency (in %)
Sepedi	50%
Tshivenda	15%
Xitsonga	20%
Other	15%



REPRESENTING DATA AS FRACTIONS AND PERCENTAGES IN PIE CHARTS

To represent data in a pie chart, you need to know how to convert (change) the frequencies of the different categories into a fraction or percentage of the total.

1. The learners in Class A were asked how many languages they could speak. The table shows the data that was collected.
   (a) Complete the 'Fraction' column by determining what fraction of the whole each category is.
   (b) Complete the 'Percentage' column by converting the fraction to a percentage.

Remember, to convert a common fraction to a percentage you have to multiply by 100%.

Number of languages spoken by learners in Class A

Languages	Frequency	Fraction	Percentage
One language	10	$\frac{10}{40} = \frac{1}{4}$	25%
Two languages	20		
Three languages	6		
Four languages	2		
More than four languages	2		
Total	**40**	$\mathbf{\frac{40}{40}}$	**100%**

   (c) Draw a pie chart of the data in your completed table. Use a circular object to draw the circle. Then estimate the sizes of the various slices of the pie chart.

2. The learners in Class B were asked how many languages they could speak. The table shows the data that was collected.
   (a) Complete the 'Fraction' column by determining what fraction of the whole each category is.
   (b) Complete the 'Percentage' column by converting the fraction to a percentage.

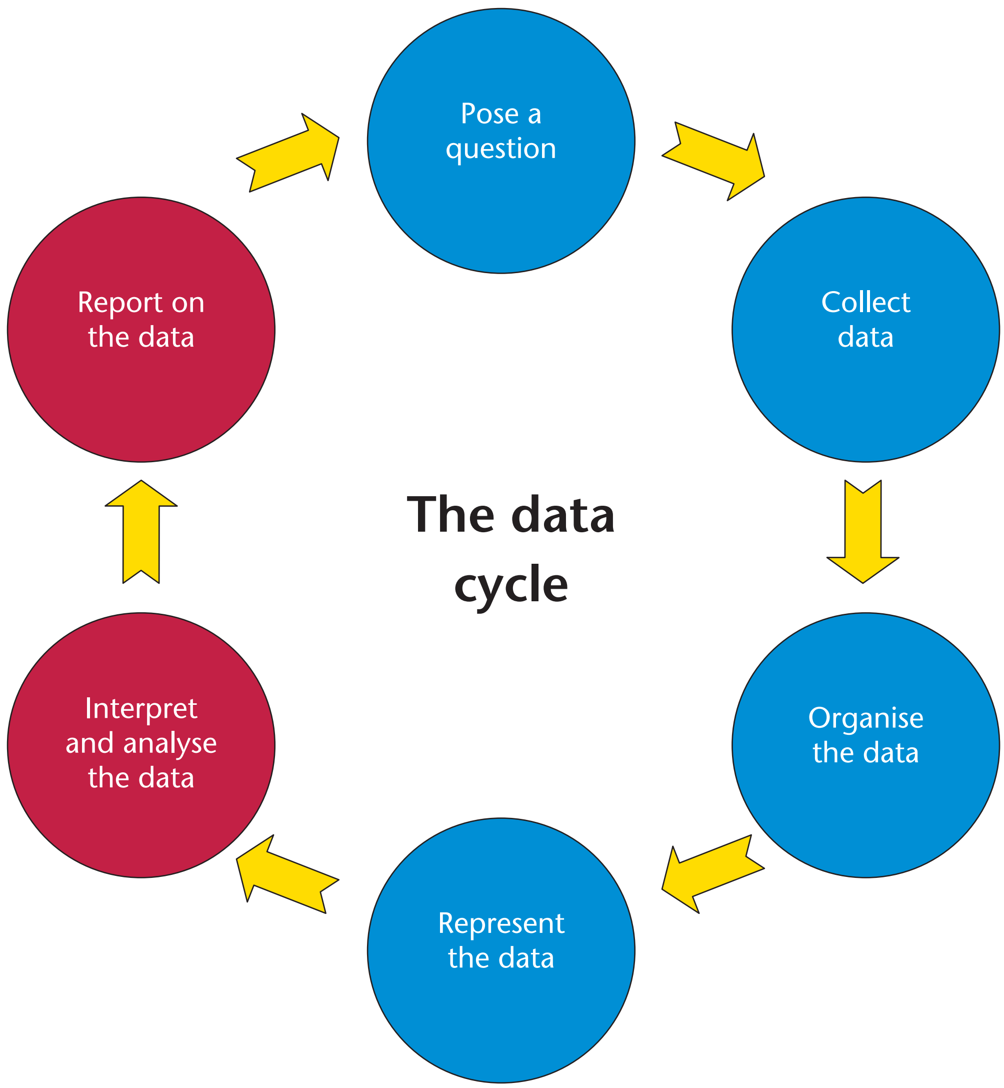
Number of languages spoken by learners in Class B

Languages	Frequency	Fraction	Percentage
One language	12	$\frac{12}{60} = \frac{1}{5}$	20%
Two languages	30		
Three languages	12		
Four languages	3		
More than four languages	3		
Total	**60**	$\mathbf{\frac{60}{60}}$	**100%**

   (c) Draw a pie chart to represent the data in your completed table.

Chapter 15
Interpret, analyse and report on data

By now you should be able to read and interpret data represented in words, bar graphs, double bar graphs, pie charts and histograms. The activities in this chapter will give you more practice in interpreting and analysing such data. At the same time, you will be asked to think critically about the data, especially how the ways in which data is presented can mislead the reader into drawing inaccurate conclusions. You will also practise reporting on data by writing short paragraphs to summarise the data presented to you.


Pose a question
Collect data
Organise the data
Represent the data
Interpret and analyse the data
Report on the data
The data cycle

15 Interpret, analyse and report on data

15.1 Interpreting and reporting on data

CRITICALLY READING AND REPORTING ON DATA

1. Read the following paragraph and answer the questions that follow.

> In 2009, a sample of 2 500 schools from about 26 000 schools across South Africa took part in a survey to provide data about learners and schools. The sample included schools from each province as follows: 415 schools from the Eastern Cape, 238 from the Free State, 265 from Gauteng, 386 from KwaZulu-Natal, 326 from Limpopo, 248 from Mpumalanga, 129 from the Northern Cape, 275 from North West and 218 from the Western Cape.
>
> Adapted from: *Census @ School Results 2009*, Statistics South Africa

(a) What was the population of the survey? ....

(b) What was the sample of the survey? ....

(c) Which province were most of the schools from? ....

(d) Which province were the fewest schools from? ....

(e) Complete the first two columns of the table by listing the provinces in order from the province that had the most schools to the province that had the fewest schools participating in the survey.

Province	Number of schools	Percentage of all schools

(f) Complete the last column by working out the percentage of the whole that the schools in each province make up. You may use your calculator for this question. (Round off to one decimal place.)

(g) Write three to five lines as a summary report of the data described in the paragraph on the previous page. The summary should give an idea of the highest and lowest data items, as this indicates the range of the data.

....

....

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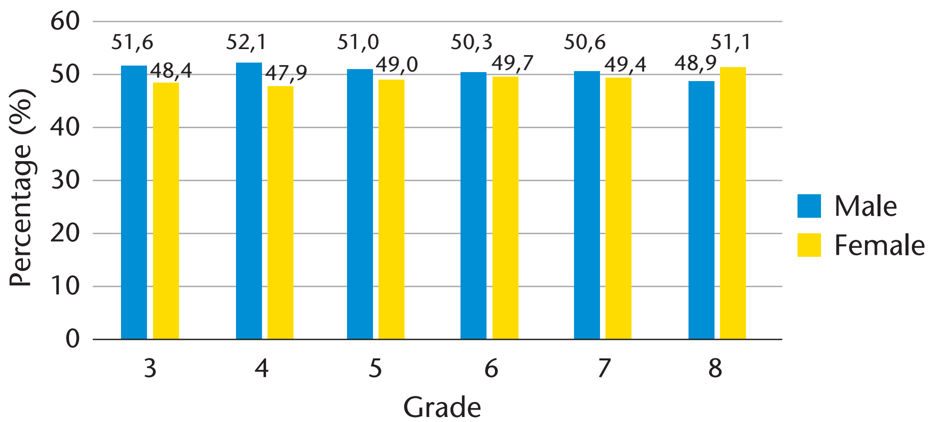
....

2. The graph below shows the percentage of male and female learners at schools in Grades 3 to 8 in 2009.

Percentage of male and female learners in Grades 3 to 8



(Source: *Census @ School Results 2009*, Statistics South Africa)

(a) Which grade has the highest percentage of females? ....

(b) Which grade has the lowest percentage of females? ....

(c) Which grade has the highest percentage of males? ....

(d) Which grade has the lowest percentage of males? ....

(e) If 150 000 Grade 6 learners took part in the survey, how many girls and how many boys were there in Grade 6? You may use your calculator.

....

....

(f) Complete the following summary report:

The graph shows that the number of male learners seems to (decrease/increase) the higher the grade. For example, in Grade 3, ... % learners were male compared to ..... % in Grade 8. The number of female learners seems to (decrease/increase) the higher the grade. For example, in Grade 3, ... % learners were female compared to ..... % in Grade 8.

(g) Based on the graph, would you expect there to be more or fewer males in Grade 10? Explain your answer.

....

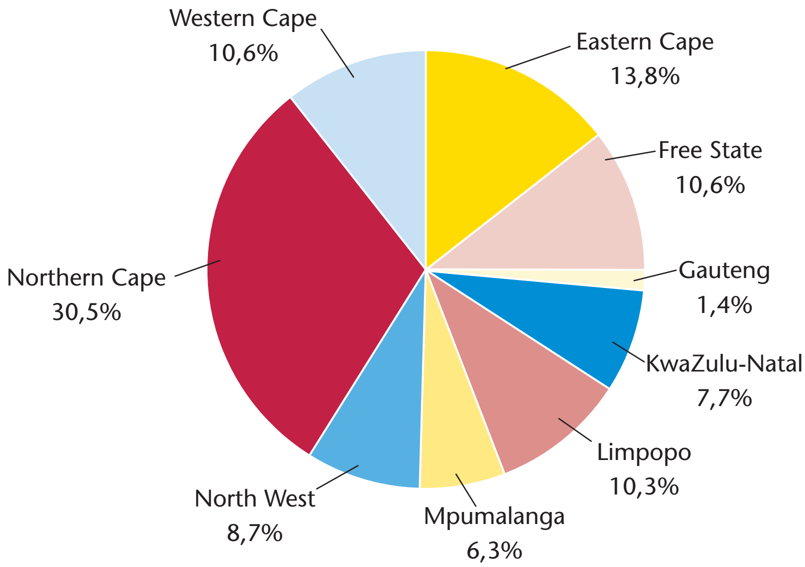
....

(h) Based on the graph, would you expect there to be more or fewer females in Grade 10? Explain your answer.

....

....

3. The following pie chart shows the land area of each province in 2011.



(Source: *Census 2011: Census in brief*, Statistics South Africa)

(a) Which province has the largest land area? ....

(b) Which province has the smallest land area? ....

(c) Which three provinces have more or less the same land area?

....

(d) How much bigger is the Northern Cape than Gauteng? (Use a calculator.)

....

(e) Are we able to tell from the pie chart which province has the largest population? Explain your answer.

....

....

....

....

(f) If the total land area of South Africa is 1 200 000 km^2, how many square kilometres are the largest and the smallest provinces?

....

....

(g) Write a short paragraph to summarise the data shown in the pie chart.

....

....

....

....

....

....

15.2 Identifying bias and misleading data

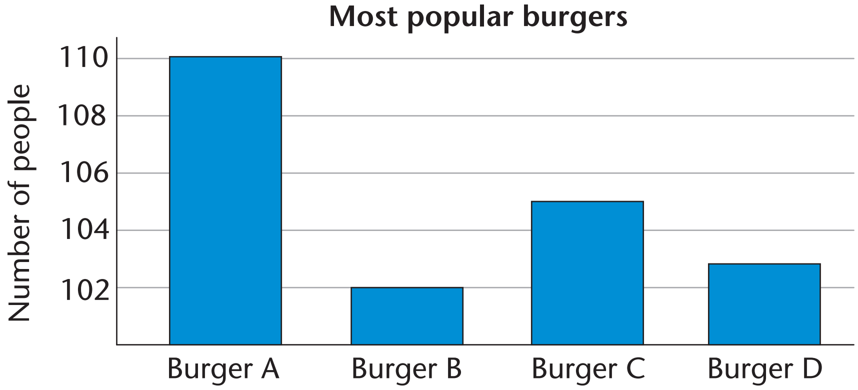
Sometimes the ways in which data is presented could be intentionally or unintentionally **biased** or misleading. As you work through the following activities, think carefully about:

- data that is not necessarily shown by the graph
- when, how and where the data was collected
- which scales are used on the graphs
- which summary statistics (mean, median and mode) are used to summarise the data.

Bias means that a person prefers a certain idea and possibly does not give equal chance to a different idea.

CRITICALLY ANALYSING DATA

1. Look at the bar graph below and answer the following questions:



   (a) Which burger is the clear favourite? ....

   (b) The height of the bars indicate that burger A is liked by five times as many people as burger B. Is this true? Look at the vertical scale.

....

....

....

....

....

....

   (c) In your exercise book, redraw the bar graph, but show the full vertical scale.

2. Look at the pie chart.

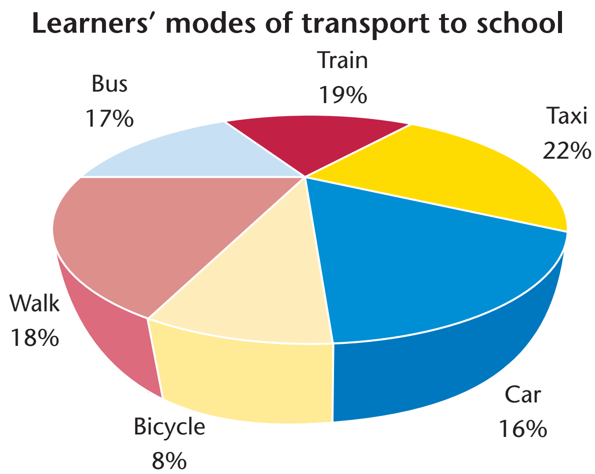
   (a) What is the second most common mode of transport that learners use?

....

   (b) Which mode of transport is the least common one?

....

   (c) Is the pie chart misleading in any way? Explain.



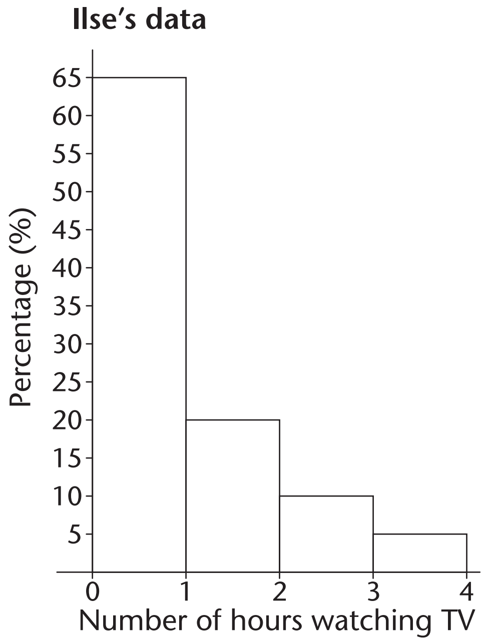
....

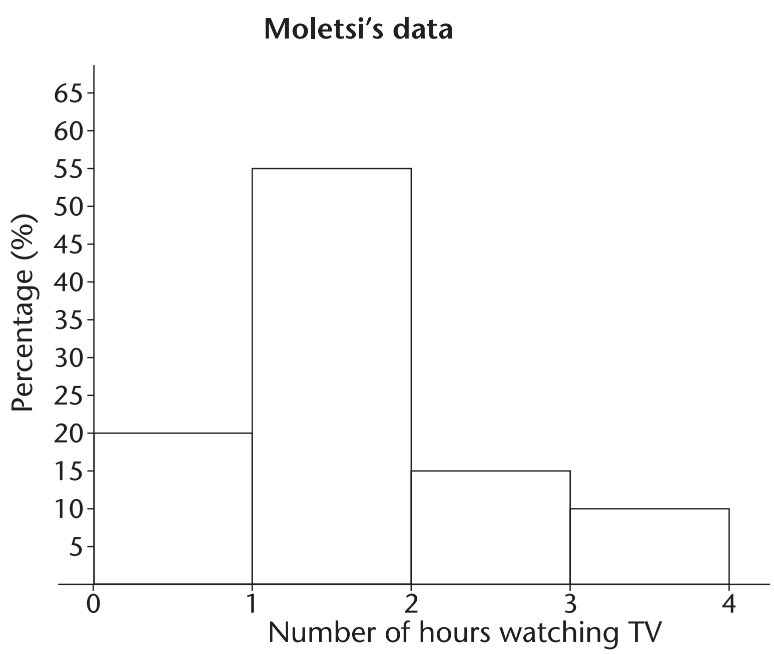
....

....

....

3. Ilse and Moletsi wanted to find out more about the number of hours people spend watching TV on a particular public holiday. Ilse did her survey on the public holiday from 13:00 to 15:00. She visited a supermarket and asked adult respondents to complete her questionnaire. Moletsi did his survey on the same day from 17:00 to 19:00. He went from door to door in his neighbourhood and asked the children to complete his questionnaire.





(a) According to Ilse's data, how long did most people spend watching TV on the public holiday?

....

(b) According to Moletsi's data, how long did most people spend watching TV on the public holiday?

....

(c) Write a paragraph to summarise and compare Ilse's data and Moletsi's data.

....

....

....

....

....

....

(d) How could the time when the data was collected have affected the data?

....

....

....

....

....

(e) How could the place where the data was collected have affected the data?

....

....

....

....

....

(f) How could the people from whom data was collected have affected the data?

....

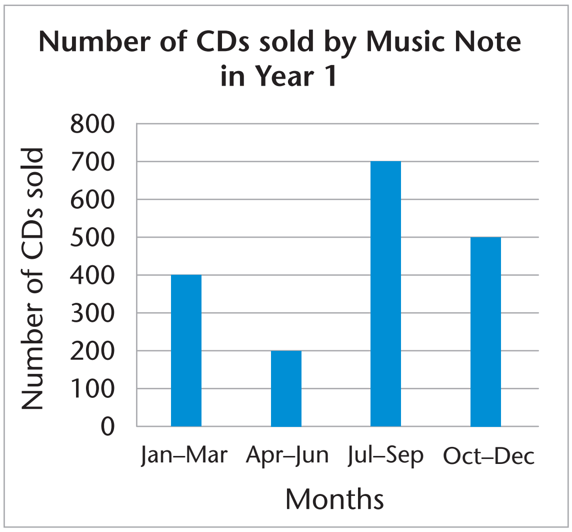
....

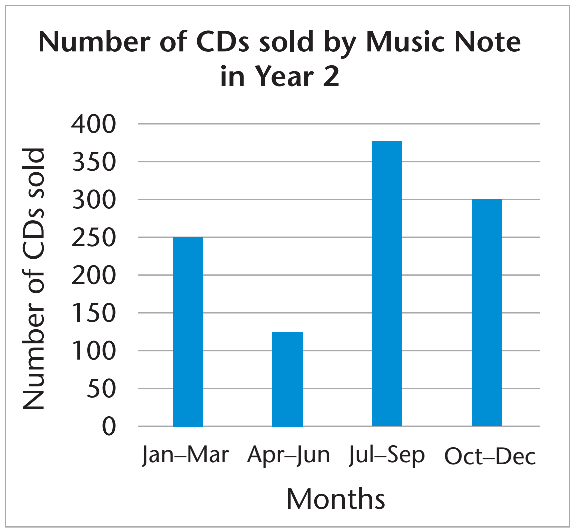
....

....

....

4. Look at the following graphs and answer the questions that follow:





(a) What does each of the graphs show?

....

....

(b) How many CDs were sold in July to September of Year 1? ....

(c) How many CDs were sold in July to September of Year 2? ....

(d) The heights of the bars indicate that Music Note sold more CDs in October to December of Year 2 than in the same months of Year 1. Is this the case?

....

....

(e) How many CDs were sold altogether in Year 1?

....

(f) How many CDs were sold altogether in Year 2?

....

(g) Explain why the heights of the bars seem to indicate that Music Note sold more or less the same number of CDs in both years, which is not true.

....

....

....

....

....

5. The following table shows the Mathematics marks of Class A and Class B.

Class A	94, 42, 23, 67, 67, 68, 13, 53, 44, 34, 64, 69, 50, 31, 91, 40, 10, 30, 49, 61
Class B	74, 26, 65, 45, 71, 77, 58, 35, 39, 45, 68, 45, 57, 62, 29, 55, 23, 56, 38, 36, 50, 64, 58, 32, 42

(a) Find the range of each set of data.

Class A: .... Class B: ....

(b) What can you say about the two classes by looking at the range of marks?

....

(c) Calculate the mean (average) Mathematics mark for each class. You may use your calculator.

Class	**Total marks**	**Number of marks**	**Mean**
Class A			
Class B			

(d) Compare the two sets of data using the means.

....

....

(e) Find the median for each class.

Class	**Marks from highest to lowest**	**Middle position**	**Median**
Class A			
Class B			

(f) Compare the two sets of data using the medians.

....

....

(g) Find the mode for each class.

Class	**Highest frequency**	**Mode**
Class A		
Class B		

(h) Compare the two sets of data using the mode.

....

....

(i) Which of the following do you think best represents each set of data: mean, median or mode? Explain your answer.

....

....

....

....

CHAPTER 16
Probability

Probability theory deals with situations that can have many possible outcomes, only one of which actually occurs. For example, when you throw a dice, only one face shows – but any of the others could have shown. Or when you cross a street you usually get to the other side without being struck by a car – but it could have happened.

1	1	4	4	3	2	4	1	3
2	5	3	3	2	4	1	2	4
3	4	1	5	4	5	3	3	2
1	5	1	4	1	1	6	5	1
4	3	6	5	5	2	3	3	4
1	3	1	5	4	5	5	3	4
3	4	4	2	4	3	3	1	4
2	6	3	3	4	4	3	4	1
1	3	6	3	2	5	6	2	4

4	5	4	1	1	2	3	1	6
4	5	4	1	2	6	2	2	5
3	1	1	2	1	4	6	1	1
2	6	2	5	6	1	3	2	2
3	5	5	3	2	1	5	4	4
3	1	4	4	1	5	2	2	1
6	1	2	2	1	3	4	3	6
6	6	6	4	5	2	4	2	1
5	2	3	1	6	3	5	5	1

4	4	3	1	5	5	5	3	2
5	2	4	6	2	5	4	5	5
2	4	2	1	5	6	2	4	4
2	4	1	1	4	5	1	3	3
4	5	2	6	6	2	4	4	2
6	2	4	1	1	6	5	5	5
1	5	5	4	1	1	2	3	2
6	4	4	2	5	3	4	4	5
5	6	2	1	3	1	2	2	3

1	3	2	2	6	1	3	2	2
1	5	6	1	5	5	1	6	4
6	4	6	2	1	6	2	6	3
3	1	4	2	5	5	2	3	4
2	1	3	4	4	4	2	2	1
2	3	2	4	6	6	3	6	5
2	4	1	5	3	2	3	4	6
1	5	3	4	3	3	6	1	5
3	5	1	2	2	3	4	6	1

4	4	4	4	3	4	6	3	3
6	4	5	4	2	2	6	6	3
1	6	6	4	4	5	3	2	6
4	1	5	2	5	3	6	4	3
2	4	3	2	2	2	6	4	2
4	2	1	5	4	3	3	1	4
3	2	5	2	6	2	5	1	5
2	2	1	3	3	3	5	5	5
5	4	4	3	3	6	6	2	2

3	5	3	6	4	6	6	3	1
3	1	1	3	5	3	2	2	1
6	4	3	1	5	1	1	1	1
5	3	2	2	6	3	4	1	1
4	2	1	3	2	5	6	3	6
6	4	4	4	1	6	1	3	3
3	4	3	2	3	1	1	1	6
1	1	3	6	2	3	1	3	3
2	2	5	6	1	1	2	3	5

6	5	4	6	6	1	5	2	2
5	1	6	3	4	4	4	2	4
2	5	1	3	6	4	2	2	2
3	1	1	5	1	6	3	4	2
2	1	5	5	5	4	1	2	2
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3	1	1	5	1	6	3	5	4
4	5	4	6	2	2	2	2	6

4	4	6	2	2	4	1	1	4
5	3	3	5	5	4	2	4	3
3	2	1	3	1	4	1	1	6
1	3	5	1	6	1	3	3	5
4	5	2	1	6	5	5	1	2
1	2	6	2	4	1	4	4	1
6	2	5	5	2	3	4	2	3
1	2	6	1	6	5	4	6	6
6	5	6	1	1	1	1	4	5

1	3	2	4	5	5	6	4	2
5	1	3	3	1	2	5	2	5
3	1	6	1	3	1	3	5	1
6	4	3	1	4	1	4	1	6
2	1	3	5	3	4	4	5	1
1	1	3	5	1	3	6	6	4
5	4	4	2	6	1	2	3	4
4	1	5	1	2	1	6	1	1
4	3	3	2	1	3	3	5	3

2	4	2	1	4	6	4	2	6
2	6	4	2	4	4	5	2	2
3	2	2	2	3	6	3	2	4
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5	4	2	5	6	2	1	5	3
3	1	1	2	1	2	5	5	1

6	1	5	3	5	2	2	5	4
5	5	4	5	1	4	1	4	2
5	6	5	4	6	4	1	4	6
2	1	5	4	1	3	1	2	6
5	5	1	1	4	6	1	1	5
3	3	2	2	4	3	5	2	4
4	6	5	1	6	1	4	3	3
6	3	5	5	2	2	5	6	1
6	5	1	6	3	3	6	6	4

1	5	4	5	5	4	5	3	4
6	3	1	2	6	3	3	4	6
1	6	6	1	2	1	5	1	6
5	2	2	4	1	3	4	6	3
2	1	2	6	2	3	5	2	2
4	5	1	2	3	3	2	5	6
2	6	6	4	2	3	6	2	2
3	6	1	5	6	3	4	4	4
1	2	3	2	2	4	6	6	4

6	1	4	2	3	2	4	6	4
2	5	3	1	2	4	1	5	4
3	4	2	5	4	5	2	3	2
1	5	1	4	1	1	6	5	1
4	3	6	5	5	3	5	3	4
1	5	1	5	4	5	5	3	2
3	4	4	2	4	3	3	1	4
1	6	2	3	6	1	6	3	5
4	5	1	2	1	2	3	4	5

3	5	4	1	6	2	5	1	6
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5	4	2	5	6	2	6	1	1
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3	5	2	3	2	1	5	4	4
3	1	4	4	1	5	2	2	1
6	1	5	2	1	4	4	3	6
5	2	6	2	4	1	6	2	1
1	6	3	3	2	6	6	4	5

4	5	2	1	1	3	5	3	2
3	2	4	4	2	5	4	5	5
2	4	2	1	5	5	2	4	4
3	6	1	2	1	3	4	3	5
4	5	2	6	6	2	4	4	2
3	2	4	1	5	6	5	5	5
1	5	5	4	5	1	2	6	2
2	1	3	5	3	3	6	6	4
5	5	5	2	1	2	2	1	2

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16 Probability

16.1 Possible and actual outcomes, and frequencies

WHAT CAN YOU EXPECT?

You will soon do an experiment. To do the experiment you need a bag like a plastic shopping bag or a brown paper bag. You also need three objects of the same size and shape, like three buttons, bottle tops or small square pieces of cardboard. The three objects must look different, for example they should have different colours such as yellow, red and blue. If you use cardboard squares, you can write "yellow", "red" and "blue" on them.

1. (a) Put your three objects in your bag. You will later draw one object out of the bag, without looking inside. Can you say whether the object that you will draw will be the yellow one, the blue one or the red one?

   ....

   (b) Discuss this with two classmates.

2. (a) Now draw an object out of the bag, write down its colour, and put it back.

   ....

   (b) You will soon do this 12 times. Can you say how many times you will draw each of the three colours? If you think you can, write your prediction below.

   ....

   ....

   (c) Compare your predictions with two classmates.

   (d) Can you think of any reason why you may draw blue more often than red or yellow, when you do the experiment described in (b)?

   ....

   ....

3. (a) Draw an object out of the bag, write down its colour, and put it back. Do this 12 times and write down the colour each time.

   ....

   ....

   (b) Write your results in the table below.

Outcome	Yellow	Red	Blue
Number of times obtained			

What you did in question 3 is called a **probability experiment**. Each time you drew an object out of the bag, you performed a **trial**.

Each time you performed a trial, three different things could have happened. These are called the **possible outcomes**.

Each time you performed a trial, one of the possible outcomes actually occurred. This is called the **actual outcome**.

The number of times that a specific outcome occurred during an experiment is called the **actual frequency** of that outcome.

4. (a) What were the possible outcomes in the experiment that you did in question 3?

   ....

   (b) How many trials did you perform in the experiment? ....

   (c) What was the actual outcome in the third trial that you performed?

   ....

   ....

   (d) What was the actual frequency of drawing a blue object during the 12 trials in the experiment that you did? ....

16.2 Relative frequencies

Thomas also did the experiment in question 3 on page 221 but he performed more trials and his results were as follows:

Outcome	Yellow	Red	Blue
Number of times obtained	5	7	8

1. (a) How many trials did Thomas perform in total? ....

   (b) What fraction of the trials produced yellow as an outcome?

   ....

   (c) What fraction of the trials produced red as an outcome?

   ....

   (d) What fraction of the trials produced blue as an outcome?

   ....

The fraction of the trials in an experiment that produce a specific outcome is called the **relative frequency** of that outcome.

$$\text{Relative frequency of an outcome} = \frac{\text{number of times the outcome occurred}}{\text{total number of trials}}$$

A relative frequency can be expressed as a common fraction, as a decimal or as a percentage. The relative frequencies in the results of the experiment Thomas did (question 1) were one quarter for yellow, 7 twentieths for red and 2 fifths for blue. Expressed as percentages, the relative frequencies were 25%, 35% and 40%. The **range** of Thomas's relative frequencies, expressed as percentages, is 15% (40% – 25%).

2. (a) Use your calculator to calculate the relative frequencies that you obtained for the three different outcomes in the experiment you did in question 3 on page 221. Express them both as fractions and percentages.

....

....

(b) Calculate the range of the relative frequencies of the three outcomes for the results of the experiment you did in question 3.

....

(c) You will soon repeat the experiment with 3 possible outcomes and 12 trials that you did. Do you think you will get the same results than when you first did the experiment? ....

3. (a) Join with three or four classmates to work as a team, and discuss question 2(c).

(b) Assign the "names" A, B, C, D and E (if you are 5) to the team members and complete the table below for the experiment you did in question 3. Give the relative frequencies as percentages. Note that to calculate the relative frequencies for the totals as percentages, you have to use your calculators.

	Actual frequencies			Relative frequencies %			Range
	Yellow	Red	Blue	Yellow	Red	Blue	
Experiment 1 by A							
Experiment 1 by B							
Experiment 1 by C							
Experiment 1 by D							
Experiment 1 by E							
Totals for experiment 1							

(c) Which of the ranges is the smallest? ....

16.3 More trials and relative frequencies

WHAT HAPPENS WHEN YOU CONDUCT MANY TRIALS?

1. Join up with your teammates of the previous session. Each of you will soon repeat the experiment you did previously. You will put a yellow object, a red object and a blue object in a bag, draw one object and note the colour. You will do this 12 times. This will be experiment 2.
   (a) Do you expect that the results will in some ways be the same as for the experiment in which you did this in the previous section? Do not talk to your teammates yet. Form your own opinion, and also consider *why* you think the results will be different or the same.

   ....

   ....

   ....

   (b) Share your ideas with your teammates.

You will soon repeat the experiment and write the results in the rows for "experiment 2" on the table on the next page. You will repeat it once more and write the results in the rows for "experiment 3". If you have time left, you may repeat it once more as "experiment 4".

2. (a) Look at the table on the next page. Certain rows are for the outcomes that you and your teammates obtain. The shaded rows are for adding different sets of outcomes together. Think about what may happen and predict in what rows the ranges will be smaller than in other rows, and in what row the range will be the smallest of all.

   ....

   ....

   ....

   (b) Share your ideas with your teammates.

3. (a) Copy the totals for "experiment 1" into the first row of the table on the next page. Do the experiment described in question 1 and enter the results in the rows for "experiment 2". Calculate the relative frequencies and the range.
   (b) Add in the results of your teammates, add up the totals and calculate the relative frequencies and the range of the totals.

4. Repeat question 3, and enter the results in the rows for "experiment 3".

		Actual frequencies			Relative frequencies %			Range
		Yellow	Red	Blue	Yellow	Red	Blue	
1	Totals for experiment 1							
2	Experiment 2 by A							
3	Experiment 2 by B							
4	Experiment 2 by C							
5	Experiment 2 by D							
6	Experiment 2 by E							
7	Totals for experiment 2							
8	Totals for experiments 1 and 2 combined							
9	Experiment 3 by A							
10	Experiment 3 by B							
11	Experiment 3 by C							
12	Experiment 3 by D							
13	Experiment 3 by E							
14	Totals for experiment 3							
15	Totals for experiments 1, 2 and 3 combined							
16	Experiment 4 by A							
17	Experiment 4 by B							
18	Experiment 4 by C							
19	Experiment 4 by D							
20	Experiment 4 by E							
21	Totals for experiment 4							
22	Totals for experiments 1, 2, 3 and 4 combined							

Term 4
Revision and assessment

Revision

Do not use a calculator for any of the questions in this section. Show all your steps of working.

INTEGERS

1. Write down the next 3 terms of the following sequences:
   (a) 3; 2; 1; 0; ... .
   (b) –10; –12; –14; ... .
2. Fill in either <, > or = to make the statement true:
   (a) –10 ☐ –11 (b) –8 + 2 ☐ –8 – 2
3. Rewrite the following numbers in order from smallest to largest: 3; –3; 0; –6; 4.

   ....
4. Write down the values of all integers that are bigger than –12 and smaller than –8.

   ....
5. Calculate the following:

(a) –4 + 5	(b) 7 – 3	(c) 8 + 4
....	....	....
(d) 5 – (–3)	(e) –2 + (–4)	(f) –1 – 2 – 3 – 4
....	....	....
(g) –40 – (–24)	(h) –24 – (–40)	(i) –13 – 15
....	....	....

6. If I descend from 10 m *above* sea level to 5 m *below* sea level, how many metres have I descended?

   ....
7. If a submarine that is 15 m below sea level rises 7 m, how far below sea level would the submarine still be?

   ....

NUMERIC PATTERNS

1. Write down the first four terms of a sequence that fits the description given:
   (a) The sequence starts at –14, and each term is 3 bigger than the previous term.

   ....

   (b) The sequence starts at 5, and each term is 4 less than the previous term.

   ....

2. Describe in words the relationship between the terms in the sequence. Then use the relationship to find the next 3 terms in the sequence.
   (a) –90; –94; –98; ...

   ....

   ....

   (b) –4; –5; –9; –14; ...

   ....

   ....

3. (a) Complete the table.

x	0	1	2	3	4	5	6	7	8
$2 - 4x$	2		–6						

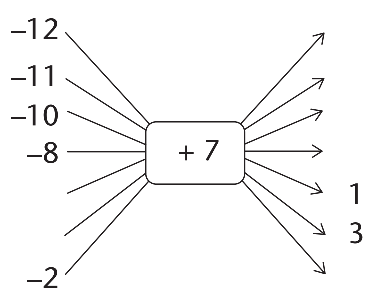
   (b) Do the output values form a pattern with a constant difference? If so, what is the constant difference?

   ....

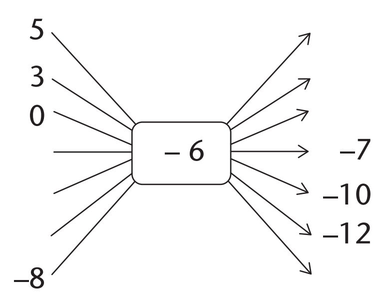
FUNCTIONS AND RELATIONSHIPS 2

1. Study the flow diagrams and fill in all the missing numbers:

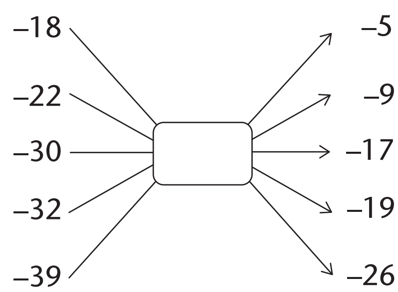
(a)



(b)



2. Fill in the rule.



3. Study the tables and fill in all the missing numbers:

(a)

Term number	1	2	3	4		8
Value of the term	–9	–12	–15			

(b)

Term number	1	2	3	4		7		
Value of the term	5	0	–5					–50

ALGEBRAIC EXPRESSIONS 2

1. Write numbers in the boxes to make the statements true:

   (a) If $x = 5$, then $x - 12 =$ ☐

   (b) If $x = -4$, then $x + x =$ ☐

   (c) If $x = -2$ and $y = 4$, then $x - y =$ ☐

   (d) If $x = -8$ and $y = -3$, then $x - y =$ ☐

2. Estelle is visiting Paris in July, and she SMSes Sonja back in Pretoria that the temperature at 6 a.m. is 13 °C. Sonja SMSes back that the temperature in Pretoria is $13 - x$ where x is 15 degrees. What is the temperature in Pretoria?

   ....

3. Consider the following situation and identify the variable quantities and the constants: A camping site charges R50 as entrance fee for a vehicle and thereafter R60 per night. The Brinks use the formula $60x + 50$ to calculate the total cost if they want to camp for x nights.

   ....

   ....

ALGEBRAIC EQUATIONS 2

1. Solve for x:

   (a) $x - 6 = -2$ (b) $x + 4 = -2$ (c) $x + x = -8$

   ....

   ....

2. Here is an equation: $2 + c = d$

   (a) Write down a pair of integers that makes the equation true. One of the integers should be positive and the other negative.

   ....

   (b) Write down a pair of negative integers that makes the equation true.

   ....

3. You are given that $x - 6 = -15$. Write down the value of:

   (a) $x - 9$ (b) $x + 2$

   ....

   ....

4. If $d = c - 5$, calculate the value of c when d has a value of -2.

   ....

   ....

COLLECT, ORGANISE AND SUMMARISE DATA

1. Write down whether each of the described groups represents a sample or a population:

   (a) the learners of Star Primary ....

   (b) 30 drivers of Toyota Corollas ....

2. Is the sample chosen for each study appropriate? Why do you say so?

   (a) The study is "favourite music amongst teenagers" and the sample is all teenagers in Grade 7 at your school.

   ....

   ....

   ....

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(b) The study is "what food should the school tuck shop stock" and the sample is 6 learners in each of the grades at your school.

....

....

....

3. Is the following multiple-choice question a good one? If it is not good, explain why and re-write it.

How old are you?

(a) 6–10 years old ☐

(b) 10–15 years old ☐

(c) 15–20 years old ☐

(d) 20+ years old ☐

....

....

....

4. The learners of one Grade 7 class at Star Primary were asked how many pets they had, and this is the data that was generated:

0; 3; 0; 1; 2; 3; 2; 1; 0; 0; 1; 3; 2; 2; 1; 1; 0; 1; 1; 2; 1; 4; 0; 1; 2; 1; 3; 0

(a) Summarise this data in a tally and frequency table.

Number of pets	Tally	Frequency

(b) Write down the modal number of pets. ....

(c) Write down the range of the number of pets. ....

(d) Determine the mean number of pets, correct to 1 decimal place.

....

....

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5. The following stem-and-leaf display represents the amount of money (in rands) in some learners' purses/wallets:

0	0	3	4	7	9		
1	0	2	4	7	9		
2	0	1	2	8			
3	0	2	2	7	9		
4	0	2	6	7	7	8	9
5	3	4					

Key: 3|2 means 32

(a) How many learners were sampled? ....

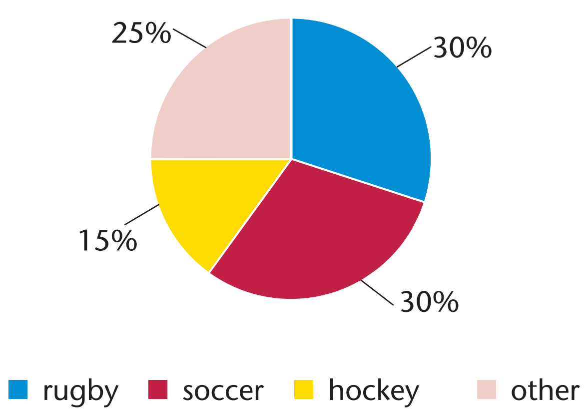
(b) What is the maximum amount of money in the purses/wallets? ....

(c) Determine the median of the amount of money in the purses/wallets.

....

REPRESENT DATA

1. The 120 learners in Grade 7 at a boys' school were asked to name their favourite winter sport, and the data was represented by the pie chart below.



(a) What percentage of the boys like hockey the most? ....

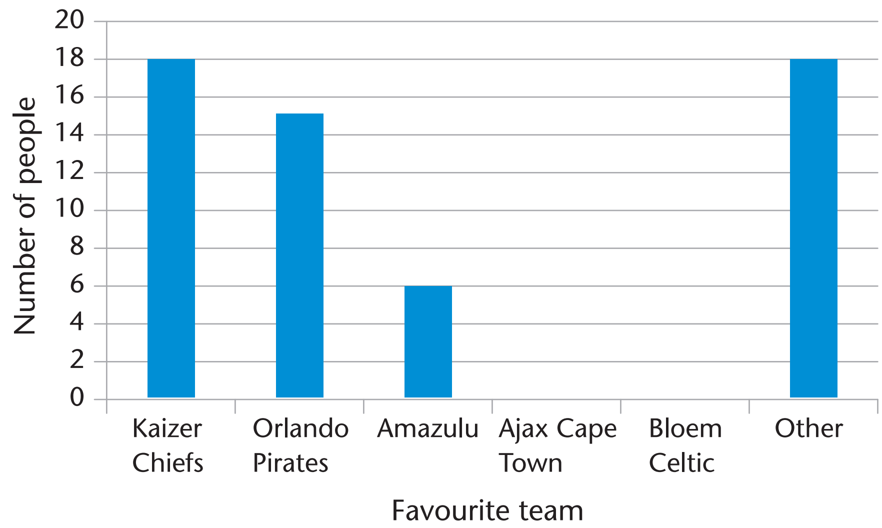
(b) How many boys like soccer the most?

....

(c) How many boys like hockey the most?

....

2. Seventy-five people were asked to name their favourite South African soccer teams. The bar graph shows the results, with two bars missing. The number of people that like Ajax Cape Town or Bloemfontein Celtic the most are equal. Draw in the missing bars.



3. Draw a histogram to show the ages of a sample of people, as recorded in this frequency table:

Age group (years)	< 20	20–29	30–39	40–49	50–59	60–69
Frequency	5	7	6	8	4	3

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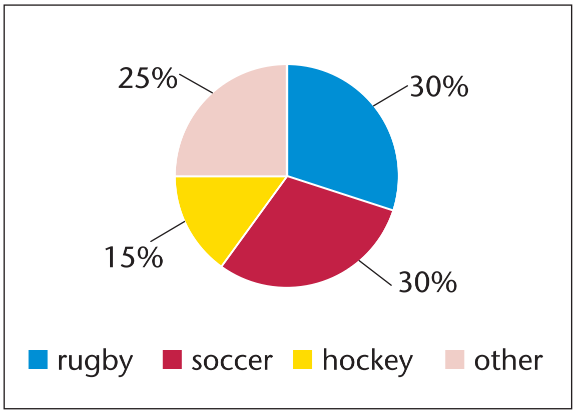
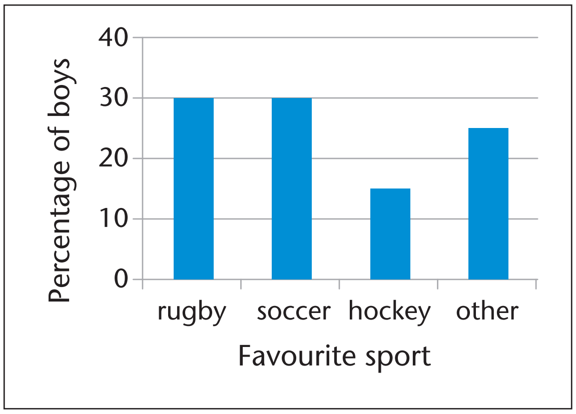
INTERPRET, ANALYSE AND REPORT ON DATA

1. The information collected from boys about their favourite winter sport (see question 1 on page 233) could be represented in a number of different ways, for example by a pie chart or in a bar graph.





What type of graph do you think is best suited to represent the data? Why do you say so?

....

....

....

....

2. Jan wants to find out how important the following activities are in people's lives: shopping, playing sport, watching TV, and doing hobbies. He goes to the shopping mall near his house and asks 20 teenagers to rank these activities in order of importance. Describe some possible sources of bias in sourcing and collecting the data.

....

....

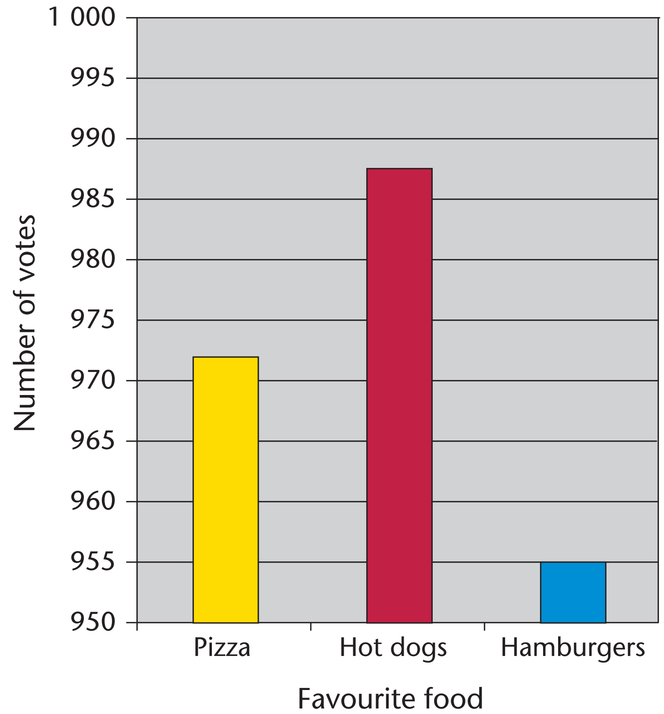
....

....

....

....

3. A survey of favourite fast foods was undertaken. The data that was collected is represented in this bar graph:



David says, "The graph shows clearly that hamburgers are by far the least favourite fast food." Do you agree with David? Explain your answer.

....

....

....

....

....

4. The following data is collected for a project. It represents the number of times nine different people have accessed their Facebook page in the past week:

5; 8; 8; 10; 15; 17; 18; 23; 59.

Which measure of central tendency (the mode, mean or median) will best represent the 'average' of the data? Explain your answer.

....

....

....

PROBABILITY

1. Rendani has 12 T-shirts: 3 are black; 4 are white; 2 are blue and 3 are green. If, one morning, he picks a T-shirt at random from his cupboard, what is the probability (given as a fraction in simplest form) that:

   (a) he chooses a white T-shirt? ....

   (b) he chooses a T-shirt that is not blue? ....

2. In a bag there are only black, green and yellow counters. You are going to take one counter out of the bag at random. If you are told that:

   - the probability that it will be black is less than a third, and
   - the probability that it will be green is twice the probability it will be black

   write down one example of how many counters of each colour there might be in the bag.

   ....

   ....

   ....

3. You are going to throw two dice at the same time, and the outcome will be the sum of the numbers showing on each dice. For example, if a 3 and a 4 is thrown the outcome will be 7.

   (a) List all the possible outcomes of the experiment.

   ....

   ....

   ....

   ....

   ....

   ....

   ....

   (b) Write down the probability the outcome will be 2.

   ....

   (c) Write down the probability the outcome will be 10.

   ....

Assessment

In this section, the numbers in brackets at the end of a question show the number of marks that the question is worth. Use this information to help you determine how much working is needed.

The total number of marks allocated to the assessment is 60.

1. Write the numbers below in order from smallest to largest. (2)
   1; –4; 0; –2; 4

   ....

2. List all the integers that are between –17 and –22. (2)

   ....

3. Calculate the following: (4)

   (a) 8 – 10 ....

   (b) 5 + (–4) ....

   (c) –2 – 3 – 4 ....

   (d) –3 – (–2) ....

4. If the temperature at sunset is 12 °C, and it drops by 16 degrees overnight, what is the temperature at dawn? (2)

   ....

5. Fill in either <, > or = to make the statement true. (2)

   (a) –1 234 ☐ –1 235

   (b) 2 – (–1) ☐ 1 – (–2)

6. Describe in words the relationship between the terms in the sequence. Then use the relationship to find the next 3 terms in the sequence. (6)

   (a) –81; –77; –73; ...

   ....

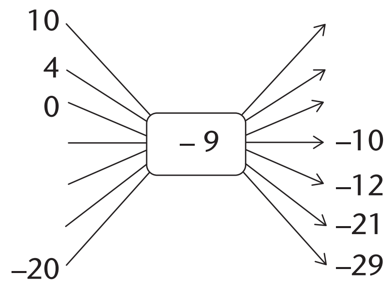
   ....

   (b) 10; 4; 6; –2; 8; ...

   ....

   ....

7. Study the flow diagram and fill in all the missing numbers: (3)



8. Write numbers in the boxes to make the statements true. (4)

(a) If $x = 10$, then $x - 14 =$ ☐

(b) If $x = -2$, then $x + x =$ ☐

(c) If $x = -1$ and $y = 7$, then $x - y =$ ☐

(d) If $x = -5$ and $y = -1$, then $x - y =$ ☐

9. Solve for x. (3)

(a) $x - 9 = -2$ (b) $x + 7 = -5$ (c) $x + x + x = -6$

....

....

10. Three possible solutions are given in brackets next to the equation, but only one is correct. Which one is correct? (1)

$-5 - x = 10$ {–5; –15; 15}

....

....

11. Here is an equation: $e + f = -5$

(a) Write down a pair of integers that makes the equation true. One of the integers should be positive and the other negative. (2)

....

(b) Write down a pair of negative integers that makes the equation true. (2)

....

12. An inter-schools shot-put competition was held and the best throws (in metres) were recorded as a stem-and-leaf display:

5	0	1	1	1	1	3	9
6	5	6	7	7	7	9	9
7	1	4	5	7	9		
8	1	1	6				
9	0	0	2	7	7	8	

Key: 5|1 means 5,1

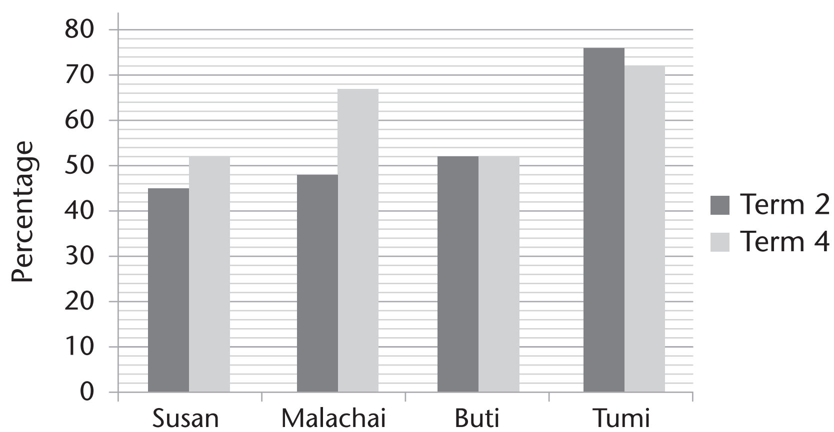
(a) How many shot-putters were there in the competition? .... (1)

(b) Only those able to throw 7 m or more in the first two throws were allowed to continue in the competition, to throw a further three times.
How many competitors were permitted to continue? .... (1)

(c) What is the modal distance of the throws? .... (1)

(d) What is the range? .... (1)

13. The graph shows the Mathematics results of four learners in two exams: the Term 2 exam and the Term 4 exam.



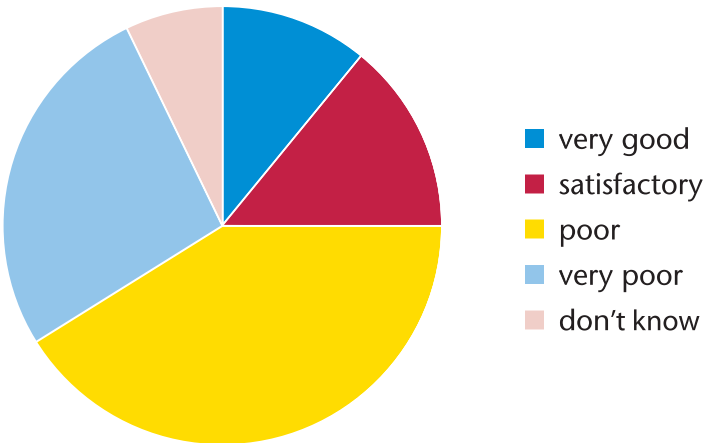
(a) What is the name of this type of graph? .... (1)

(b) Whose results remained constant? .... (1)

(c) Who performed best in Term 4, and what was his or her percentage? (2)

....

14. In a survey, 200 people were asked about the quality of services they received from their local municipality. The bar graph summarises their responses.



(a) Which response was the most common? .... (1)

(b) About how many people thought the services received were either very good or satisfactory? (2)

....

(c) Siyoli says that about 80 people think the services received were poor. Is Siyoli right or wrong, or is it impossible to tell from the information provided? (1)

....

(d) If you had been the person responsible for this survey, what feedback would you give the local municipality? (1)

....

15. Ashwell wanted to collect information on which fast food was the most popular amongst 12- to 13-year-olds. He collected information by asking 10 of his friends which fast food they liked the most.

Discuss any problems with Ashwell's process of data collection, and suggest better alternatives. (4)

....

....

....

16. You are going to throw two dice at the same time, and the outcome will be the product of the numbers shown on the dice. For example, if a 3 and a 4 is thrown the outcome will be 12.

(a) List all the possible outcomes of the experiment. (3)

....

....

....

....

....

(b) Write down the probability that the outcome will be 6.

Give your answer as a fraction in simplest form. (3)

....

17. A bag contains 20 counters of 3 different colours. I am going to take one counter from the bag, without looking.

(a) Fill in the missing information in this table: (3)

Colour of counters	Number of counters	Probability of choosing this colour
red	5	
white		$\frac{1}{4}$
blue	10	

(b) Suppose that before taking a counter out of the bag, I add an extra white counter to the bag. How will this affect the probability that I will take a red counter?

Tick one of these options: (1)

- It will increase the probability. ☐
- It will decrease the probability. ☐
- The probability will remain the same. ☐
- It is not possible to tell. ☐